Review for Final Exam – Chapter 11

11.1 Curves defined by parametric equations. To describe a curve in the plane, you can either give y as a function of x, as you usually did in Calculus I and II, or you can give both y and x as functions of a third variable t. The variable t is called a *parameter.* Sometimes it is useful to think of t as a time variable, and the functions $x = f(t)$ and $y = g(t)$ as the functions which tell you where you are in the plane at a given time. Notice that a single curve in the plane can be described in many different ways by parametric equations; just as there are many different ways for a moving point to travel along the curve: cf. Examples 2 and 3.

Re-read this section from the beginning through Example 4. Read also Example 6.

11.2 Calculus with parametric curves. Suppose we are given parametric equations for a curve, which means we are given equations for x and y as functions of t. We can use these functions and their derivatives dx/dt and dy/dt to find the slope of the curve at a given point, the concavity of the curve at a given point, the area under the curve between two given points, the length of a curve between two given points, and the surface area obtained by revolving a part of the curve around an axis.

Formulas for doing this are obtained by starting from the corresponding formulas we learned in Calculus I and II, which involve dy/dx and d^2y/dx^2, and using the chain rule to replace derivatives with respect to x by derivatives with respect to t. Thus, the formula for area we learned in Calculus II, $A = \int y\ dx$, becomes $A = \int y(dx/dt)dt$.

Finding d^2y/dx^2 can be a little tricky when y and x are given parametrically. Review the class notes, problem 1 on Quiz 1, and problems 11 to 16 on p. 702.

Read this entire section.

11.3 Polar coordinates. The first few pages of the chapter are especially important for people who hadn't seen much of polar coordinates before taking this class. Make sure you understand everything through Example 8 very clearly. You have to be pretty up-to-date on your basic trigonometry to do well in this part of the course; see the review of trigonometry available on the course web page for a list of the things you need to know. Also read the sections titled "Symmetry" and "Tangents to Polar Curves" on pp. 710–712. You can skip the section on graphing polar curves with graphing devices.

11.4 Areas and lengths in polar coordinates. Read the entire section carefully. It introduces two formulas: the formula for area in the boxes on p. 716 and the formula for arc length in the box on p. 718. When using the formula for area, $A = \int_a^b \frac{1}{2}r^2 d\theta$, you have to be clear about what numbers to use for a and b in order to capture the area you are trying to compute. Here is where it's important to have a good background in graphing in polar coordinates, as described in examples 6, 7, 8 of Section 11.3.

11.5 Conic sections. Psychologists distinguish between *working memory* and *long-term memory.* Your working memory, which is very limited in size, is where you store things that are new to you while you are in the process of reasoning about them and comparing them to things you already know. After you process and organize new information, it can go into your long-term memory, which is, as far as we can tell, unlimited in size. In order to do well on solving a test problem, you want to leave your working memory as unencumbered as possible, so it is free to work efficiently with the new material presented in the test problem. That means you should put as much as possible in your long-term memory, which you can draw on effortlessly while working on the problem.

The point of all this is simply that if you're doing a problem involving conic sections, you'll have a better chance of solving it if you have already learned by heart which which kind of equations give ellipses and which give hyperbolas, so you don't have to waste part of your brainpower paying attention to things like that. I would suggest memorizing the material in the red boxes numbered 1, 4, and 7 in this section, in connection with the figures that go along with them. I do not recommend, however, memorizing boxes 5 or 8, because the idea is to memorize things in an organized fashion. If you memorize boxes 4 and 7, you only have to switch the roles of x and y to get the results in boxes 4 and 8.

Although I'm recommending memorization here, it is still important to understand where the formulas in these boxes come from. Why, for example, are the foci of an ellipse at the points $(\pm c, 0)$ where $c^2 = a^2 - b^2$? Where does the formula $c^2 = a^2 - b^2$ come from? The better you understand the answers to questions like this, the better you will organize and retain what you memorize.

To be honest, I'm not going to ask you to remember much of this material about conics on the final exam. But whatever you learn from this section will be of significant help to you in understanding later material, including the quadric surfaces which appear in section 13.6 and in a lot of examples in chapters 15, 16, and 17, which are covered in Calculus IV.

11.6 Conic sections in polar coordinates. The same comments apply here as to the preceding section. Try to memorize what's in the box on p. 729. (One thing that's left out of the box is that the *directrix* of the conic is given by $x = \pm d$ or $y = \pm d$, depending on which equation is involved. See figure 2 in the margin.) I might, at most, ask a question like those in exercises 9 through 16 at the end of this section.