Review for the Final Exam

The final exam is comprehensive. To prepare for it, you can use the review sheets for the first three exams (you can find them at the course web page), along with the suggestions below for studying the material that came after the third exam.

You should memorize the following: the definition of the definite integral, the proof that the derivative of e^x is e^x, the proof that the derivative of $\ln x$ is $1/x$, the proof that the derivative of $\arcsin x$ is $1/\sqrt{1-x^2}$, and the proof that the derivative of $\arctan x$ is $1/(1+x^2)$. You can find more details about this definition and these proofs in the review sheets for the first three exams. Notice, by the way, that of the four proofs listed above, the last three are similar to each other in that they rely on the same basic idea: first express x as a function of y and then find dy/dx by using implicit differentiation.

One comment about finding volumes of solids of revolution by integration. Students often have trouble figuring out which formula to use on a given problem. I recommend starting each one of these problems by drawing a thin rectangle in the region that you are going to revolve around the axis of revolution, and figuring out what the correct formula is for the object (disc, washer, or shell) that you get by revolving this little rectangle. After you do this you will automatically know what to integrate. Notice in particular that if the thin rectangle is thin in the x-direction, then its base will be of length dx and the integral will be an integral with respect to x; if the thin rectangle is thin in the y-direction, then its height will be dy and the integral will be an integral with respect to y.

Finally, I recommend memorizing the integrals numbered 1 through 18 in the Table of Integrals at the back of the book (page 6 in the blue Reference Pages). I am not likely to ask a question where you have to remember some of the trickier ones, like number 15 or number 18, but by memorizing these you will get a feel for which integrals are the basic ones. (When you are given a complicated integral and trying to reduce it to a simpler one, you need to know which ones the simpler ones are, so you know what to aim for.) Also, notice that the integrals numbered 12, 13, 16, 17, and 18 can be done by making a substitution which reduces them to one of numbers 1 through 11. So you really don't need to memorize numbers 12, 13, 16, 17, and 18, as long as you know how to figure them out by yourself.

8.7 Approximate integration. Although this is an important topic, it's not suitable for exam questions when calculators are not allowed. There won't be a question on this topic on the final exam.

8.8 Improper integrals. Read from the beginning of the section through Example 8.

9.1 Arc length. Read from the beginning of the section through Example 2. Notice that there are two different formulas for arc length, given in Equations 3 and 4 in the red boxes on page 585. Both of these can be remembered at the same time by noticing that they can both be written in the form

$$L = \int \sqrt{(dx)^2 + (dy)^2}.$$

To get the formula in Equation 3 on page 585, just factor $(dx)^2$ out from under the square root sign; to get the formula in Equation 4, factor $(dy)^2$ out from under the square root sign.

9.2 Area of a surface of revolution. Read the entire section. Here there are four formulas for arc length, but you can remember them easily by noticing that for surfaces obtained by revolving a curve around the y-axis, you use

$$S = \int 2\pi x \sqrt{(dx)^2 + (dy)^2},$$

and for surfaces obtained by revolving a curve around the x-axis, you use

$$S = \int 2\pi y \sqrt{(dx)^2 + (dy)^2}.$$

As in the formula for arc length, each of these two formulas can be turned into either an integral with respect to x (by factoring $(dx)^2$ out from under the square root sign) or an integral with respect to y (by factoring $(dy)^2$ out from under the square root sign).

9.3 Applications to physics and engineering. Read from the beginning of page 600 through Example 6. Formulas for the coordinates $\bar{x}$ and $\bar{y}$ of the centroid of an area are given in Equation 8 on page 603 and Equation 9 on page 604.