$\boxed{3.1.15}$ Show $\lim\left(\frac{n^2}{n!}\right) = 0.$

Proof: Let $\varepsilon > 0$

There exists $K \in \mathbb{N}$ s.t. $\boxed{K > \frac{1}{\varepsilon} + 3}$.

If $n \geq K$, then $n > \frac{1}{\varepsilon} + 3$, so $n - 3 > \frac{1}{\varepsilon}$, so $\frac{1}{n-3} < \varepsilon$.

Also $\frac{n^2}{n!} = \frac{n \cdot n}{n(n-1)(n-2)(n-3)\cdots 4\cdot 3\cdot 2\cdot 1} = \frac{n}{(n-1)(n-2)(n-3)\cdots 4\cdot 3\cdot 2\cdot 1}$

$\leq \frac{n}{(n-1)(n-2)}$.

So $\frac{n^2}{n!} \leq \frac{n}{(n-1)(n-2)}$. But $\frac{n}{(n-1)(n-2)} = \frac{n}{n^2-3n+2} \leq \frac{n}{n^2-3n} = \frac{1}{n-3}$.

So $\frac{n^2}{n!} \leq \frac{1}{n-3}$. Hence $\frac{n^2}{n!} < \varepsilon$,

so $\left|\frac{n^2}{n!} - 0\right| < \varepsilon$. $\boxed{QED}$

$\boxed{3.1.16}$ Show $\lim\left(\frac{2^n}{n!}\right) = 0$

Proof: Let $\varepsilon > 0$

There exists $K \in \mathbb{N}$ s.t. $\boxed{K > \frac{4}{\varepsilon}}$

If $n \geq K$, then $\frac{4}{n} < \varepsilon$.

Also $\frac{2^n}{n!} = \frac{2\cdot 2\cdot 2\cdots 2\cdot 2\cdot 2\cdot 2}{n(n-1)(n-2)\cdots 4\cdot 3\cdot 2\cdot 1}$ (where there are n factors of 2 in the numerator)

$\leq \left(\frac{2}{n}\right)\left(\frac{2}{n-1}\right)\left(\frac{2}{n-2}\right)\cdots\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{2}{2}\right)\cdot\left(\frac{2}{1}\right)$

$\leq \frac{2}{n}\cdot 1\cdot 1 \cdots 1\cdot 1\cdot 1\cdot 2 = \frac{4}{n}$

So $\frac{2^n}{n!} \leq \frac{4}{n}$.

Hence $\frac{2^n}{n!} < \varepsilon$

So $\left|\frac{2^n}{n!} - 0\right| < \varepsilon$ $\boxed{QED}$