EXAM 3
Math 2423
4-30-10

Name ______________________

Row ____________

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(16 points)*

a) Find the derivative of the function $f(x) = e^{\arctan(x^3)}$.

6

$$e^{\arctan(x^3)} \cdot \frac{1}{1+(x^3)^2} \cdot 3x^2$$

2 2 2

b) Use logarithmic differentiation to find the derivative of the function $f(x) = (\ln x)^{(x^2)}$. Show your work.

10

$$y = (\ln x)^{(x^2)}$$

$$\ln y = \ln\left((\ln x)^{(x^2)}\right) \quad 2$$

$$\ln y = x^2 \ln(\ln(x)) \quad 1$$

$$2 \quad \frac{1}{y}\frac{dy}{dx} = 2x \cdot \ln(\ln(x)) + x^2 \frac{1}{\ln(x)} \cdot \frac{1}{x} \quad 4$$

$$\frac{dy}{dx} = \left(2x \ln(\ln(x)) + \frac{x}{\ln x}\right) \cdot y \; 1 = \left(2x(\ln \ln x) + \frac{x}{\ln x}\right)$$

2. *(20 points)* Find the integrals, showing all work. Remember to express your answer as a function of x.

a) $\int \frac{\sin x}{1+\cos^2 x}\, dx$

10

2 $u = \cos x$

2 $du = -\sin x \, dx$

$$= \int \frac{-du}{1+u^2} = -\arctan u \quad 2$$

2

$$= -\arctan(\cos x) + C$$

2

b) $\int \sin^{10} x \cos^3 x \, dx$ =

10

1 $\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$
1

-1 $du = -\cos x dx$

2

know formula

application

$$\int \sin^{10} x \, (\cos^2 x) \cos x \, dx$$

2 -1 $\sin^2 x - 1$

-1 $1 + \sin^2 x$

$$= \int \sin^{10} x \, (1 - \sin^2 x) \cos x \, dx \quad 1$$

$$= \int u^{10}(1-u^2)\, du = \int [u^{10} - u^{12}]\, du$$

$$= \frac{u^{11}}{11} - \frac{u^{13}}{13} + C = \frac{(\sin x)^{11}}{11} - \frac{(\sin x)^{13}}{13} + C$$

2 1

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3. *(20 points)* Use L'hopital's rule to find the limits.

[10] a) $\lim_{x\to\infty}\frac{\ln x}{x^2}$ $= \lim_{x\to\infty}\frac{\frac{1}{x}}{(2x)} = \lim_{x\to\infty}\frac{1}{2x^2} = 0$

$\left(\frac{\infty}{\infty}\right)$ 2 2 2 2 2

[10] b) $\lim_{x\to 0}\frac{x-\sin x}{x^3}$ $= \lim_{x\to 0}\frac{1-\cos x}{3x^2} = \lim_{x\to 0}\frac{+\sin x}{6x} = \lim_{x\to 0}\frac{\cos x}{6} = \frac{\cos 0}{6} = \frac{1}{6}.$

2 2 2 2

$\left(\frac{0}{0}\right)$ 2 $\left(\frac{0}{0}\right)$ $\left(\frac{0}{0}\right)$

4. *(20 points)* Find the integrals, showing all work.

10) a) $\int x\sin 3x\,dx$ $= \int u\,dv = uv - \int v\,du$ 2 $= x\left(-\frac{\cos 3x}{3}\right) - \int\left(-\frac{\cos 3x}{3}\right)dx$

$\boxed{\begin{array}{ll} 1\ u = x & dv = \sin 3x\,dx \ 1 \\ du = dx & v = -\frac{\cos 3x}{3} \ 2 \\ 1 & \end{array}}$

$= -\frac{x\cos 3x}{3} + \frac{1}{3}\int \cos 3x\,dx$ 1

$= -\frac{x\cos 3x}{3} + \frac{1}{3}\,\frac{\sin 3x}{3} + C$ 2

[10] b) $\int x^8\ln x\,dx$ $= \frac{x^9\ln x}{9} - \int\frac{x^9}{9}\cdot\frac{1}{x}\,dx$ 2

$\boxed{\begin{array}{ll} u = \ln x \ 1 & dv = x^8 \ 1 \\ du = \frac{1}{x}\,dx \ 1 & v = \frac{x^9}{9} \ 1 \end{array}}$

$= \frac{x^9\ln x}{9} - \frac{1}{9}\int x^8\,dx$ 2

$= \frac{x^9\ln x}{9} - \frac{1}{9}\cdot\frac{x^9}{9} + C$ 2

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5. *(24 points)* Find the integrals, showing all work. Remember to express your answer as a function of x. You may need to use the formula $\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta|$.

a) $\int \frac{1}{\sqrt{1+x^2}}\,dx$

$$= \int \frac{1}{\sqrt{1+\tan^2\theta}} \sec^2\theta \, d\theta = \int \frac{1}{\sec\theta} \sec^2\theta \, d\theta$$

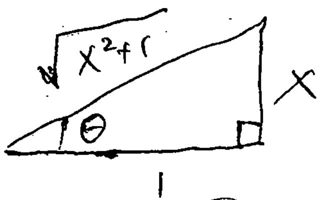
1 $x = \tan\theta$

1 $dx = \sec^2\theta \, d\theta$

1 1 1 1

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$



$$= \ln\left|\sqrt{x^2+1} + x\right| + C$$

2 2

2 b) $\int \frac{\sqrt{1-x^2}}{x^2}\,dx$ (Hint: make the substitution $x = \cos\theta$.)

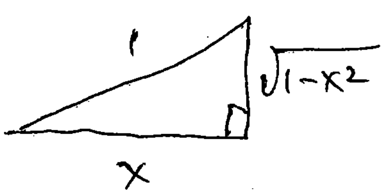
$$= \int \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta}(-\sin\theta \, d\theta)$$

1 $x = \cos\theta$

$dx = -\sin\theta \, d\theta$

1

$$= \int \frac{\sin\theta(-\sin\theta)}{\cos^2\theta} \, d\theta$$



1

1

$$= -\int \frac{\sin^2\theta}{\cos^2\theta} \, d\theta = -\int \tan^2\theta \, d\theta$$

1

$$= -\int(-1+\sec^2\theta)\,d\theta = \int(1-\sec^2\theta)\,d\theta$$

1 1

$$= -\tan\theta + \theta$$

$$= -\frac{\sqrt{1-x^2}}{x} + \arccos \cancel{x} + C$$

1 1