Trigonometry Review

Although you probably know enough about trigonometry to get by in this class already, it might make the class a lot easier for you if you take a little time to brush up on the following basic facts. They're taken from Appendix D of the text, but you don't need to memorize everything that's in Appendix D — only what's mentioned below.

Radian measure of angles. We don't use degree measures for angles at all in calculus if we can help it, the reason being that the formulas that you've learned for derivatives and integrals of trigonometric functions are actually incorrect when degree measures are used. (If we define a function $f(x)$ by setting $f(x)$ equal to the sine of the angle whose measure is x degrees, then the derivative of this function is *not* the cosine of the angle whose measure is x degrees.) So try to get used to expressing all angles in radian measure. When you want to convert from degrees to radians, remember that 360 degrees is 2π radians — so you have to multiply degrees by $2\pi/360$, or $\pi/180$, to get radians.

I still use degree measure often, because it's just easier to refer to angles that way. But as soon as derivatives or integrals enter the picture, it's imperative to switch to radian measure!

Definition and values of trigonometric functions. For angles θ between 0 and 90 degrees, you can define the trigonometric functions by drawing a right triangle with angle θ in it, and letting $\sin\theta$ be the opposite side over the hypotenuse and $\cos\theta$ be the adjacent side over the hypotenuse (see figure 6 on page A26 in Appendix D). Then you can figure out the values of the other four trig functions by using the facts that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}. \tag{1}$$

You should memorize the side lengths of the 45-45-90 triangles and 30-60-90 triangles (Figure 8 on page A26). With those memorized you then automatically know the values of all the trigonometric functions at $\theta = \pi/6$, $\pi/4$, and $\pi/3$.

To find the sines and cosines of angles θ of more than 90 degrees, or less than 0 degrees, you have to use the definitions

$$\cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{y}{r}. \tag{2}$$

Here x and y are the x and y coordinates of a point P which lies on the line through the origin which makes an angle θ with the positive x-axis; and r is the distance from P to the origin. (It's often easiest to take $r = 1$, so P is on the unit circle.) Once you know the sine and cosine, you can find the values of the other trigonometric functions using the formulas in equations (1) above. The procedure of how to use these formulas is illustrated in Example 3 on page A27.

You can use the above definition and your knowledge of the 45-45-90 and 30-60-90 triangles to easily figure out the sine and cosine of any angle which is a multiple of 30 or 45 degrees (including negative angles). You don't need to memorize tables like the one in blue on page A27.

Quite often you have to find the angles at which the sine and cosine take the values 0, 1, and -1. The easiest way to do this is by using the formulas in (2). For example, to find the values of θ for which $\cos\theta = -1$, you think of the point P on the unit circle whose x-coordinate is -1; that is, the point $P = (-1, 0)$. This is on a line which makes an angle of 180 degrees, or π radians, with the positive x-axis, so the cosine of π is zero. But the same line also makes other angles with the positive x-axis: namely, $\pi \pm 2\pi$, $\pi \pm 4\pi$, etc. So the angles whose cosine are zero are: π plus or minus any multiple of 2π. In other words, $\cos\theta = 0$ if θ is in the set

$$\{\ldots, -7\pi, -5\pi, -3\pi, -\pi, \pi, 3\pi, 5\pi, 7\pi, \ldots\}.$$

Trigonometric identities. You should know the basic trigonometric identity,

$$\sin^2\theta + \cos^2\theta = 1. \tag{3}$$

Another identity that comes up a lot is

$$\tan^2\theta + 1 = \sec^2\theta, \tag{4}$$

but one bit of good news is that you don't have remember (4) separately, since you can always derive it from (3) when you need it, simply by dividing (3) by $\cos^2\theta$ and using the first two definitions in (1).

There are a number of other trigonometric identities in Appendix D, all of which are occasionally useful, but the only other ones I recommend you memorize are:

$$\sin(-\theta) = -\sin\theta, \qquad \cos(-\theta) = \cos(\theta)$$

and

$$\begin{aligned} \sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta. \end{aligned}$$

Any other identities you might need on the exams, I'll supply to you on the exams themselves.

Graphs of the trigonometric functions. You should memorize the graphs of the trigonometric functions (Figure 13 of Appendix D).

That completes the review of what you need to remember of trigonometry from pre-calculus. As an added bonus, here's a summary of what you will need to remember of trigonometry from calculus:

Derivatives and antiderivatives of trigonometric functions. You should memorize the formulas for the derivatives of the six trigonometric functions:

$$\begin{aligned} \frac{d}{dx}(\sin x) &= \cos x & \frac{d}{dx}(\csc x) &= -\csc x\cot x \\ \frac{d}{dx}(\cos x) &= -\sin x & \frac{d}{dx}(\sec x) &= \sec x\tan x \\ \frac{d}{dx}(\tan x) &= \sec^2 x & \frac{d}{dx}(\cot x) &= -\csc^2 x \end{aligned}$$

Another bit of good news is that you don't have to remember the corresponding antiderivative formulas separately — they're just the reverse of the above formulas. For example,

$$\text{Since } \frac{d}{dx}(\tan x) = \sec^2 x, \text{ then } \int \sec^2 x\ dx = \tan x + C.$$

One thing to watch out for is that we don't yet have formulas for the antiderivatives of $\tan x$ and $\sec x$. So you have to be careful not to inadvertently indulge in wishful thinking, and say that the antiderivative of $\tan x$ is $\sec^2 x$, or something of the sort. We will be getting formulas for the antiderivatives of $\tan x$ and $\sec x$ later, in chapter 7, after we've introduced logarithms. These won't be covered on the first exam, of course, but if you want a sneak preview, the formulas are

$$\int \tan x\ dx = \ln|\sec x| + C, \qquad \int \sec x\ dx = \ln|\sec x + \tan x| + C.$$