Putnam Seminar — Week 3

13. (B2, 2006) Prove that, for every set $X = \{x_1, x_2, \dots, x_n\}$ of n real numbers, there exists a non-empty subset S of X and an integer m such that

$$\left| m + \sum_{s \in S} s \right| < \frac{1}{n+1}.$$

14. (A5, 1997) Let N_n denote the number of ordered n-tuples of positive integers $(a_1, a_2, \dots, a_n)$ such that $1/a_1 + 1/a_2 + \cdots + 1/a_n = 1$. Determine whether N_{10} is even or odd.

15. (B1, 1988) A *composite* (positive integer) is a product ab with a and b not necessarily distinct integers in $\{2, 3, 4, \dots\}$. Show that every composite is expressible as $xy + xz + yz + 1$, with x, y, and z positive integers.

16. (B1, 1973) Let a_1, a_2, ... , a_{2n+1} be a list of $2n+1$ integers (not necessarily distinct) such that, if any one of them is removed, the remaining ones can be divided into two lists of n integers with equal sums. Prove that $a_1 = a_2 = \cdots = a_{2n+1}$.