> An otherwise empty folder screams out
> I love Putnam Graders! ❤
> Zero, reluctantly.

48 Hours of Putnam

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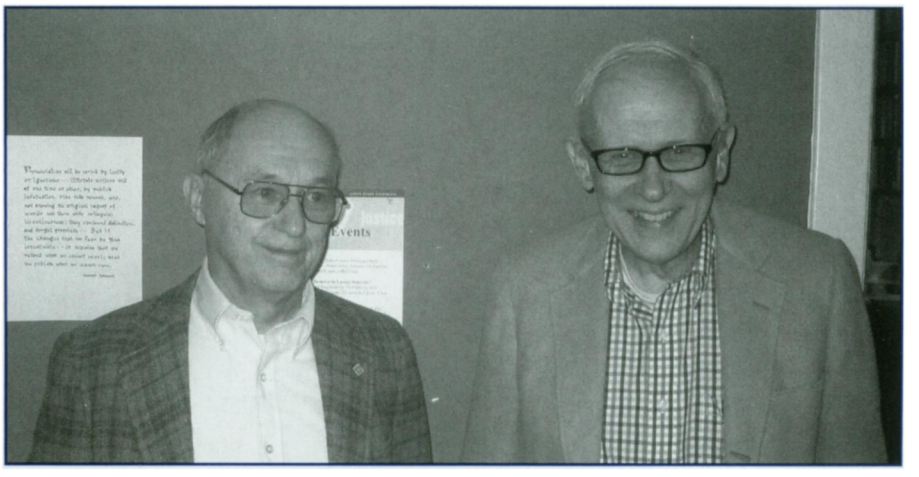
Wednesday, 8:00 p.m. The last of my freshman seminar students unceremoniously deposits his final exam on the stack, and I wish him well over the break and next semester. I tidy up the stack and put it in my carry-on bag. I head directly to my car so I can make it to Sea-Tac airport for my 10 p.m. departure, and I ask myself why, oh why, am I heading out to grade the Putnam Competition *again*? And why am I taking my *own* exams along to grade on the plane?

Wednesday, 10:30 p.m. The plane trip to San Jose is uneventful, and in addition to grading some of my own exams, I review the solutions for Problem A-5: Put the Dyck paths of length n having an even length return into one-to-one correspondence with the Dyck paths of length n - 1. I've been sent the solutions formulated by the Problem Committee, and I've scanned the `sci.math` newsgroup for alternate approaches. A short taxi ride and I'm dropped at the motel. Straight to bed, as I'm expected at 8 a.m. in O'Connor Hall on the Santa Clara University campus nearby. Experience tells me it'll be a long day tomorrow.

Thursday, 7:45 a.m. After breakfast at Denny's I meet up with some other graders who have also flown in the night before from colleges and universities all over the country. It's a pleasurable half-mile walk to campus, renewing old acquaintances from earlier years and making new ones. I was an undergraduate at Santa Clara so the Putnam grading trip is a sort of annual mathematical homecoming for me. I'm reminded of my grandfather, who *slept* in O'Connor Hall when it was a dormitory in the 1920s.

Thursday, 8:00 a.m. There are handshakes all around as the fifteen graders congregate with two of the Directors, Jerry Alexanderson and Leonard Klosinski, just outside the Sussman Room. Eventually we stop socializing and file in to search for the table with our problem number and the names of the two or three graders initially assigned to that problem. I'm pleased to find that I'm paired with an old friend from graduate school days who is also an experienced grader. Along the south wall of the room, I warily eye the cardboard boxes neatly stacked full of completed problem folders. There seem to be more this year.

Thursday, 8:04 a.m. We are just grading the A session problems; on Saturday and Sunday a similar group of graders will tackle the B session problems. Leonard Klosinski takes up his position in the doorway and with no words spoken, commands our attention. "It would appear all of you have done this before." From his notebook, he reads, "Problems should be scored as a 0, 1, 2 or 8, 9, 10. Write the score in red in the upper left-hand corner, unless the paper is written in red, then use the green pen. Do not circle the score, as it makes it look too much like a zero. Keep the folders in numerical order. If you find a particularly elegant solution, please bring it to the attention of Loren Larson." Loren Larson is the third Director, and one of his duties is collecting especially novel or unique solutions. That's about all the instruction we need, since many in the room have been coming back year after



Putnam directors Leonard Klosinski (left) and Jerry Alexanderson (right).

By the Numbers

The Putnam Competitions 1938–2003

Joseph A. Gallian
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$500: First place prize for winning team in 1938.

$25,000: First place prize for winning team in 2003.

$50: First place prize to Putnam Fellows (top five scorers) in 1938.

$2,500: First place prize to Putnam Fellows in 2003.

163: Number of participants in 1938.

3615: Number of participants in 2003.

67: Number of participants in 1946 (following World War II).

24: Number of times Harvard has won the team competition (most team wins).

8: Most consecutive years that Harvard finished first (1985-1992).

15: Most consecutive years that Harvard did not finish first (1967–1981).

9: Number of times Caltech has won the competition (second most team wins).

0: Number of team wins by Princeton, Yale, Columbia, and Stanford combined.

7: Number of times Princeton has finished second.

year. It's my seventeenth time, and there are other graders in the room who were already seasoned veterans when I was a rookie.

Thursday, 8:06 a.m. My grading partner and I talk briefly about Dyck paths and the problem, before sampling a few papers at random from our stack, in order to get a feel for the problem, possible correct solutions, and possible incorrect attempts.

Thursday, 8:15 a.m. Our quick preview has not produced anything we didn't expect. Our problem asks for a 1–1 correspondence between two sets. We decide first what we feel is essential to a complete proof, and what will therefore be required for a score of ten. We also decide how many points to award for an explicit description of the correspondence (in just one direction), but without a substantial proof (likely a well-defined inverse mapping). We'll wait for other situations to occur before we make the hard decisions about other levels of partial credit.

Thursday, 8:26 a.m. Many papers are clearly worth zero, many do not finish, some claim the correspondence doesn't exist. Many have submitted their work on the possibility that one or two points will be awarded for substantial progress toward a solution. We are reminded quickly to scan the end of the folder first. Every now and then a paper has a correct solution.

Thursday, 9:17 a.m. I finish grading a difficult solution that was correct and earned a ten, but it has taken me fifteen minutes to see that. So before diving back into the stack, I take a quick break and survey the room. The Sussman Room is much like any other departmental common room or library. A few tables, back issues of AMS and MAA journals along one wall, last quarter's tutoring schedule on a blackboard, faculty and student mailboxes in one corner, geometric models on shelves.

Today, and for the following three days, it holds fifteen graders, two or three to a table. Along one wall are the twelve stacks of completed folders for all the problems, organized in cardboard boxes tipped up on end. The stacks measure anywhere from eight inches to three feet in height.

Thursday, 9:20 a.m. Alice Kelly comes around and collects a short stack of graded papers from each grader, crisscrossing them to demarcate consecutive runs of papers. She disappears down the hall to the room with the computers, spreadsheets and cardboard boxes of graded folders. It's a mammoth job to keep all the folders collated and organized and to ensure that the scores are entered correctly, since there must be in excess of 10,000 folders submitted. Alice does a great job supervising the assistants on this front, despite her hollow threat every year to call it quits and not return the next year.

Thursday, 9:22 a.m. I get a paper where the argument is basically complete, but one of the details is wrong. Is it a minor detail? Should we call this an 8 or 9? Or might it appear to be minor, but really it is central (or equivalent) to some key step of the proof? Should it be a 2? It's time to consult with my grading partner. I feel it is a 2, my partner feels it is an 8. I argue my position, he argues his. He convinces me, and we settle on an 8. We make a note of the decision for use later, and especially for the inevitable regrading. These are the toughest decisions the graders make, and perhaps ultimately have the greatest impact on how scores are distributed.

Thursday, 9:32 a.m. Alice comes around again, but she's carrying snacks. No Putnam graders have ever gone hungry during their stay.

Thursday, 11:29 a.m. I get a paper with an especially economical presentation. Everything that needs to be said is there,

Putnam graders at work.

and there's nothing extra, no unnecessary notation, no decomposition into unnecessary case-by-case analysis, all in the space on the front side of the folder. It's pretty. After rafts of noble, yet incorrect attempts, and correct, but lengthy solutions, it's a breath of fresh air to get a pretty one. I show it to my grading partner as if it's a work of art in a coffee table book. "Look at this one!" I say, knowing he'll appreciate it. It gets the same ten all the other correct, yet lengthy, solutions get. No extra credit for style, but it's appreciated.

Thursday, Noon Klosinski appears in the doorway and says nothing. It's our cue for lunchtime. I enjoy the warmth walking over to the Faculty Club in just shirtsleeves in December. My colleagues from Southern California remark how cold it seems.

Thursday, 2:05 p.m. I get a correct, novel solution. It's not like any paper I've seen yet, or the official solutions, or the solutions posted on the Internet. I show it to my grading partner and Loren Larsen.

Thursday, 3:46 p.m. I get a paper where the student has written only an entertaining limerick.

> There once was a guy named Sam,
> who took the Putnam exam.
> He was a doof,
> and produced only one proof:
> that was no good at math (or poetry).

I'm obligated to give it a zero.

Thursday, 4:08 p.m. I see another solution, basically identical to the one I saw two hours ago for the first time. Only in reading this version, it dawns on me that it is novel only superficially, and at its core it's using the same techniques as the other "standard" solutions.

Thursday, 5:00 p.m. Klosinski signals quitting time, and we carpool into San Jose for a relaxed dinner as a group at a nice restaurant.

Friday, 8:00 a.m. We return to O'Connor Hall, and all is as we had left it. Unfortunately, the overnight janitors have not worked down our stacks.

Friday, 8:05 a.m. A student attempts to use group theory to solve our problem. It leads nowhere and gets a zero.

Friday, 8:20 a.m. I get a solution with a formula that doesn't look like anything I've seen before. But I can't quite follow the logic behind its creation. So I substitute in a few small integers, where I know what the formula should evaluate to. It fails to yield the correct answers.

Friday, 8:45 a.m. An otherwise empty folder screams out

I love Putnam Graders! ♥

Zero, reluctantly.

Friday, 10:07 a.m. I get a paper with only a huge frowny face on it. I dutifully award it a zero, but add the sad eyes

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2: Highest finish by a liberal arts school (Oberlin in 1972).

3: Number of state universities in the US that have won the team competition. (Michigan State three times, University of California at Berkeley, and University of California at Davis).

7: Number of times a Canadian institution has won the team competition.

250: Number of individuals who have been named a Putnam Fellow.

5: Number of four-time Putnam Fellows.

3: Number of perfect scores in history of the competition.

2: Number of Nobel Prize winners who were Putnam Fellows.

5: Number of Nobel Prize winners who achieved Honorable Mention or better status.

3: Number of women Putnam Fellows between 1938–2003.

2: Number of women Putnam Fellows between 2002–2003.

3: Number of freshman Putnam Fellows in 2003 competition.

67: Lowest top score between 1967–2003.

19 : Highest median score between 1967–2003.

1: Median score in 1981, 1987, 1997, 2001, 2003.

0: Median score in 1989, 1999, 2000. ■

ing technique. We require a little bit of outside work for Putnam TNG; to attend, students must have worked at least one of these problems in the week before (which they should be ready to present to the group). Students move back and forth between the two seminars that run simultaneously, depending on whether or not they have had time to attack that week's problem set.

As for the actual competition, we believe the secret to having students succeed on the Putnam Exam is the three Cs: classify, conquer and complete. First, students should look at the problems and try to classify them by the methods needed to attack them—deducing that a problem is susceptible to the Pigeonhole Principle or Strong Induction is often half the battle. Once a student has figured out which problem she wants to attack, it is time to conquer it—that is, scratch out the solution and convince oneself that all the loose ends have been nailed down. Finally, and perhaps most importantly, she needs to complete the problem; write out a carefully worded, neatly written and well-organized solution. We tell students it is better to nail down a single problem completely than to turn in multiple half-baked solutions. (For confirmation of this recommendation see page 5.)

On the day of the exam, we provide pastries and orange juice for students as they sit down for the exam. Lunch time at the dining hall is a free-for-all discussion of who got what problems. By the end of the day, everyone is exhausted, but we can usually coax students into writing up some of the solutions for our Putnam bulletin board. Students like to visit this board throughout the year, either to re-live their Putnam experience or to wistfully ponder whether they will take it next year!

Finally, we like to give students a souvenir of their grueling time together. To commemorate a particular exam, we create mugs emblazoned with a graphic associated with one of that year's Putnam problems, and give them to all students who have competed that year. Students like to collect each year's edition, and it gives them a nice reminder of a Saturday spent challenging themselves with twelve truly vexing math problems.

Our seminar accomplishes several goals. It gives students a chance to survey a variety of mathematical topics, learn problem-solving skills, and practice mathematical writing and presentation in an enjoyable social setting. We like to think that the fun of doing math in this way is the big draw, but of course, it could be the pizza... ■

A collection of problem solving resources is available at www.math.hmc.edu/putnam

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and frown to my red zero in the corner. Maybe that will liven things up down the hall where a couple of assistants are entering all the scores into huge spreadsheets and double-checking them.

Friday, 10:46 a.m. I lean back in my chair and prop open yet another folder. It's *filled* with eraser dirt, which spills out onto my lap. An occupational hazard.

Friday, 11:03 a.m. We finish grading all the solutions for A-5, as my grading partner and I load-balance our final stacks so we finish at the same time. Some others have finished their problems before us and have been assigned to help out on other problems. But first, I'll clean up and turn in our criteria for partial credit, along with some techniques that make grading certain types of solutions easier.

Friday, 11:10 a.m. We are assigned to A-1, where the need is greatest. It asks for a count of certain partitions of a positive integer. One of the two initial graders of this problem gives us a quick tutorial on techniques and common pitfalls. He'll be kept busy the next few hours as a consultant to the several graders now working on the problem.

Friday, 11:15 a.m. Alexanderson visits each team of graders, soliciting someone to regrade. All the papers for the top teams and the top individuals get a second look, a process called "regrading." This accomplishes two purposes. First, the numerical score on the folder is checked against a final copy of a spreadsheet with all the scores, providing a double-check on the data entry. But more importantly, it allows for one grader to consistently apply the awarding of partial credit to all the top competitors and to apply any lessons learned later in the grading session to papers graded earlier.

Friday, 1:40 p.m. I get a paper written in red, green, and black ink. I ask Klosinski what color pen I should use to write the score.

Friday, 3:00 p.m. A student writes, "What did one math book say to the other?" "Boy, we sure have problems."

Friday, 4:10 p.m. My plane departs at 6 p.m., so I say goodbye to everyone and Klosinski gives me a ride to the airport. Almost all the problems are finished and with the full group working on just A-1, they'll be finished within the hour, so I don't feel too guilty leaving a little early.

Friday, 7:57 p.m. My plane touches down back at Sea-Tac Airport. And I've finished grading my freshman seminar final exams on the plane ride home. ■