Some problems from *Mathematical Miniatures*

1. Consider all sequences with length $2n+1$ (where n is a positive integer) whose terms are each 0 or 1. What fraction of them have more occurrences of 1 among the last $n+1$ digits than among the first n?

2. We shall call a permutation $(x_1, x_2, \ldots, x_n)$ of $\{1, 2, \ldots, 2n\}$ pleasant if $|x_i - x_{i+1}| = n$ for at least one $i \in \{1, 2, \ldots, 2n-1\}$. Prove that, for each $n \geq 1$, more than half of all permutations are pleasant.

3. Let M be the number of integer solutions of the equation

$$x^2 - y^2 = z^3 - t^3$$

with the property $0 \leq x, y, z, t \leq 10^6$, and let N be the number of integer solutions of the equation

$$x^2 - y^2 = z^3 - t^3 + 1$$

that have the same property. Prove that $M > N$.