Problem 1

In quadrilateral $ABCD$, $\angle B$ is a right angle, diagonal $\overline{AC}$ is perpendicular to $\overline{CD}$, $AB = 18$, $BC = 21$, and $CD = 14$. Find the perimeter of $ABCD$.

Solution

Problem 2

Let set $\mathcal{A}$ be a 90-element subset of $\{1, 2, 3, \ldots, 100\}$, and let S be the sum of the elements of $\mathcal{A}$. Find the number of possible values of S.

Solution

Problem 3

Find the least positive integer such that when its leftmost digit is deleted, the resulting integer is $1/29$ of the original integer.

Solution

Problem 4

Let N be the number of consecutive 0's at the right end of the decimal representation of the product $1!2!3!4!\cdots 99!100!$. Find the remainder when N is divided by 1000.

Solution

Problem 5

The number $\sqrt{104\sqrt{6} + 468\sqrt{10} + 144\sqrt{15} + 2006}$ can be written as $a\sqrt{2} + b\sqrt{3} + c\sqrt{5}$, where a, b, and c are positive integers. Find $a \cdot b \cdot c$.

Solution

Problem 6

Let $\mathcal{S}$ be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$ where a, b, c are distinct digits. Find the sum of the elements of $\mathcal{S}$.

Solution

Problem 7

An angle is drawn on a set of equally spaced parallel lines as shown. The ratio of the area of shaded region $\mathcal{C}$ to the area of shaded region $\mathcal{B}$ is 11/5. Find the ratio of shaded region $\mathcal{D}$ to the area of shaded region $\mathcal{A}$.

- The cube immediately on top of a cube with edge-length k must have edge-length at most $k+2$.

Let T be the number of different towers than can be constructed. What is the remainder when T is divided by 1000?

Solution

Problem 12

Find the sum of the values of x such that $\cos^3 3x + \cos^3 5x = 8\cos^3 4x \cos^3 x$, where x is measured in degrees and $100 < x < 200$.

Solution

Problem 13

For each even positive integer x, let $g(x)$ denote the greatest power of 2 that divides x. For example, $g(20) = 4$ and $g(16) = 16$. For each positive integer n, let $S_n = \sum_{k=1}^{2^{n-1}} g(2k)$. Find the greatest integer n less than 1000 such that S_n is a perfect square.

Solution

Problem 14

A tripod has three legs each of length 5 feet. When the tripod is set up, the angle between any pair of legs is equal to the angle between any other pair, and the top of the tripod is 4 feet from the ground In setting up the tripod, the lower 1 foot of one leg breaks off. Let h be the height in feet of the top of the tripod from the ground when the broken tripod is set up. Then h can be written in the form $\frac{m}{\sqrt{n}}$, where m and n are positive integers and n is not divisible by the square of any prime. Find $\lfloor m + \sqrt{n} \rfloor$. (The notation $\lfloor x \rfloor$ denotes the greatest integer that is less than or equal to x.)

Solution

Problem 15

Given that a sequence satisfies $x_0 = 0$ and $|x_k| = |x_{k-1} + 3|$ for all integers $k \geq 1$, find the minimum possible value of $|x_1 + x_2 + \cdots + x_{2006}|$.

Solution

See also

- American Invitational Mathematics Examination
- AIME Problems and Solutions
- 2006 AIME I Math Jam Transcript
- Mathematics competition resources