Congruences

If n is a natural number, $a \equiv b \pmod{n}$ means that a and b leave the same remainder when divided by n.

Wilson's Theorem: If p is a prime, then $(p-1)! \equiv -1 \pmod{p}$.

Fermat's Little Theorem: If a is a natural number and p is a prime, then $a^p \equiv a \pmod{p}$.

Euler's phi function: Two numbers are said to be "relatively prime" if their greatest common divisor is 1. If m is a natural number, $\phi(m)$ is the number of natural numbers between 1 and m which are relatively prime to m. Thus, for example, $\phi(6) = 2$ because out of the numbers $\{1, 2, 3, 4, 5, 6\}$, only two (namely 1 and 5) are relatively prime to 6. Also $\phi(7) = 6$, since out of the numbers $\{1, 2, 3, 4, 5, 6, 7\}$, six of them (all but 7 itself) are relatively prime to 7. Notice that for any prime number p, obviously $\phi(p) = p - 1$.

Euler's Theorem: If a and m are natural numbers, and a and m are relatively prime, then $a^{\phi(m)} \equiv 1 \pmod{m}$

Example: Suppose p is a prime and $p \geq 7$. Show that the number $111 \cdots 1$, in which there are $p-1$ 1's, is divisible by p.