WIGGD 2025 PROJECT DESCCRIPTIONS

Project 1: An illustrated guide to Anosov flows in dimension 3.
Group leaders: Kathryn Mann (Cornell University) and Ana Rechtman (Université Grenoble Alpes)

Anosov flows are central examples of continuous dynamical systems, and there is a particularly rich structure theory for these flows on manifolds of dimension 3. As well as dynamics, many geometric and topological tools can be used to build, describe and classify these flows on 3-manifolds. Each Anosov gives a 1-dimensional foliation of the manifold by orbits, and a pair of invariant transverse 2-dimensional foliations, and classifying them amounts, in some sense, to understanding how these foliations fit together. In addition, there are rich connections with contact structures.

Much of this subject is highly visual in nature. It is often easy to understand why something is true when you see the picture, and fill in formal details from there, but difficult to glean the overarching strategy from formal proof written in explicit coordinates. The purpose of this project is to produce an illustrated guide to introductory essential material on flows, especially Anosov flows, in dimension 3. The project leaders will, ahead of time, identify important results in this area with a focus on those where the "standard" proof in the literature is incomplete or could have a simpler exposition, especially with visual aid. The group will work together to learn the required background, and then will produce concise proofs and explanations in a visual, illustrated format. As well as experience with 3-dimensional geometry/topology or dynamics, project participants will ideally have experience with or interest in at some format of illustrating mathematics (for instance, using a simple vector drawing or illustration app on a tablet/ipad).

Project 2: Cannon–Thurston maps and subgroup distortion.
Group leaders: Pallavi Dani (Louisiana State University) and Emily Stark (Wesleyan University)
A highly influential result of Cannon and Thurston proves that if M is a closed hyperbolic 3-manifold fibering over the circle with fiber a closed surface S, then the inclusion $\pi_1(S) \hookrightarrow \pi_1(M)$ extends to a continuous $\pi_1(S)$-equivariant surjective map between the visual boundaries of the groups. This yields a naturally occurring space-filling curve $S^1 \to S^2$ with finite point preimages. Mj showed that such continuous boundary extensions of group inclusions $H \hookrightarrow G$ (now called Cannon–Thurston maps) exist for much more general pairs of hyperbolic groups, for instance if H is normal or G is a particular type of graph of groups. We will explore the relationship between subgroup distortion and properties of Cannon–Thurston maps.

Project 3: End-periodic homeomorphisms and the geometry of 3-manifolds.
Group leaders: Autumn Kent (University of Wisconsin - Madison) and Marissa Loving (University of Wisconsin - Madison)
A closed 3-manifold M which admits a taut, depth one foliation can be built from gluing together so-called end-periodic mapping tori of end-periodic along their boundaries (together with some trivial product regions). These boundary components correspond to the compact leaves of the original depth one foliation while the noncompact leaves of the original foliation appear as fibers in the mapping tori. Thus, one can hope to better understand the geometry of M in terms of the combinatorics of this depth one foliation by characterizing the geometry of these mapping tori pieces in terms of their end-periodic monodromy. In this project we will explore how the volume of an end-periodic mapping torus is related to the translation length of its monodromy on an appropriate Teichmüller space. Since, in general, the Teichm̈uller space of infinite-type surfaces has not been well-studied or understood. Thus, this project will require understanding tools and techniques from the study of hyperbolic 3-manifolds as well as studying the appropriate notion of a Teichmüller space for infinite-type surfaces and how it is acted on by big mapping class groups.

Project 4: Conjugators in higher rank lamplighter groups.

Group leaders: Carolyn Abbott (Brandeis University) and Rose Morris-Wright (Middlebury College)

The conjugacy problem, a classic decision problem in group theory, asks if, given a presentation of a group, there is an algorithm to determine whether two elements of the group are conjugate. If two elements g, h in a finitely generated group G are conjugate, one can ask whether there must always exist a 'short conjugator,' that is, an element that congugates g to h and whose length is bounded by a function $f\colon \mathbb{Z} \to \mathbb{R}$ of the lengths of g and h. If such a bound exists, then the conjugacy problem is solvable in G: given two arbitrary elements $g, h \in G$, one only needs to check whether the (finitely many) elements in a ball of radius $f(|g| + |h|)$ in G conjugate g to h. The lower the complexity of the function, the faster the time of the algorithm solving the conjugacy problem.

For many groups, there is a bound on this conjugator length function: such a bound exists for hyperbolic groups, special cubical groups, mapping class groups, and many solvable groups, among others. The goal of this project is to bound the conjugator length function for the class of *higher rank lamplighter groups*, a generalization of the traditional lamplighter group $\mathbb{Z}/2\mathbb{Z} \wr \mathbb{Z}$. The Cayley graphs of higher rank lamplighter groups, which are horocyclic products of trees, have a tractable geometry which we aim to exploit to solve the problem. Sale showed that, for traditional lamplighter groups, the function f is linear, while it is polynomial for certain other wreath products. These methods will be the starting point for investigating conjugators in higher rank lamplighter groups. This project requires very little background beyond a basic understanding of group theory.