MATH 2934 Writing and Style Guide

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Contents

1 Introduction **2**

2 An Example **3**

3 Writing and Style Guide **6**

3.1 Write in Complete Sentences 6
3.2 Declare The (Distinct) Names You Give to Objects 7
3.3 Connect Logically Related Equations with $\Rightarrow$ 8
3.4 Always Include Variables on Functions 10
3.5 Support Your Claims 11
3.6 Connect Your Final Answer Back to the Problem Statement . 12
3.7 Circle Your Final Answer 14

1 Introduction

There are two main, equally important things that I would like for you to take away from this course. The first is, of course, the content itself: making sense of the concepts presented and developing the ability to solve particular kinds of problems related to those concepts.

The second, which may be entirely new to you, is communicating your knowledge of that content to others in a formal way. In practice, one does not generally do math in a vacuum, as it were. Those of you who go on to utilise this course's content may find yourselves working on engineering or scientific teams. Others might go into academic research. Wherever you end up, you will likely want or need to share your work with others, and they need to be able to interpret your writing. Thus, learning to communicate mathematics with others in a formal, standardised way is an indispensable skill.

To the latter's end, this document contains a brief summary of the most general and significant notes on writing and presentation style that I would like you to internalise.

2 An Example

To demonstrate many of the core writing techniques you should become familiar with during the semester, we begin with an example from Calculus I.

> Suppose that 20m of fencing is to be used to enclose a rectangular region. What is the maximum area that can be enclosed?

Here is one solution [my comments are contained in square brackets, and all of the footnotes are clickable links]:

> *Solution.* Let the length and width of the rectangular region be called ℓ and w, respectively [1] [2]. We are told that ℓ and w must satisfy:
>
> $$2\ell + 2w = 20$$
>
> Furthermore, we wish to maximize the area of the enclosed region. Let:
>
> $$A(\ell, w) = \ell w$$
>
> $A(\ell, w)$ is the area of a rectangular region with length ℓ and width w. We must find the maximum value of this function, subject to the constraint given above.
>
> Note first that:
>
> $$2\ell + 2w = 20 \;\Rightarrow\; w = \frac{20 - 2\ell}{2} = 10 - \ell \quad [3]$$
>
> Therefore, we have:
>
> $$A(\ell, w) = \ell w \text{ and } w = 10-\ell \;\Rightarrow\; A(\ell) = \ell(10-\ell) = 10\ell-\ell^2 \qquad [4]$$
>
> [$A(\ell, w)$ becomes $A(\ell)$ since we used the equation $w = 10 - \ell$ to eliminiate the w variable in $A(\ell, w)$]. The maximum area that can be enclosed is therefore the maximum value of this single-variable function $A(\ell)$. We now set about finding the latter.
>
> Note that:
>
> $$A'(\ell) = 10 - 2\ell$$

Therefore, we have that:

$$\begin{aligned} 10 - 2\ell = 0 &\Rightarrow 2\ell = 10 \\ &\Rightarrow \ell = 5 \end{aligned}$$

This means that, by definition [5], $A(\ell)$ has only one critical number: $\ell = 5$. Furthermore, note that:

$$A'(4) = 10 - 8 = 2 \qquad \text{and} \qquad A'(6) = 10 - 12 = -2$$

By the first derivative test, $A(\ell)$ is maximized at $\ell = 5$.

Since

$$A(5) = 5(10 - 5) = 25,$$

the maximum area which can be enclosed in a rectangular region surrounded by 20m of fencing is 25m^2.

[6] [7] $\square$

Footnotes [links to sections of this document with additional details are included in each]:

[1] Be sure to write any statements you make in complete sentences, as we have done throughout this solution.

[2] Make sure to declare the names you give to any objects with either a "Let…" statement or with the := symbol. Here we declare that the symbols ℓ and w will be used to stand in for the length and width of the rectangular region, respectively.

[3] When you use one equation or a set of equations to deduce another, use the $\Rightarrow$ symbol to connect these equations, as we do several times in this example, including here.

[4] Whenever you discuss a function or use it in calculations, include all of its variables. Notice that throughout this solution we always write $A(\ell, w)$ or $A(\ell)$, as appropriate; but we never drop all of our variables and just write A.

[5] Always aim to explain exactly why a given statement you make is so. Explain how you know what you're claiming. For example, here we tell the reader that it's the definition of a critical number, combined with our work, which allows us to conclude that $\ell = 5$ is the only critical number of $A(\ell)$.

[6] Make sure that your final answer is connected back to what the problem asked you to find. Here, the problem asked us to find the maximum area which can be enclosed in a rectangular region surrounded by 20m of fencing, so our final answer states, unambiguously, exactly what this is.

[7] Always circle your final answer, to remove any uncertainty about which piece of your work is meant to be this. Here we circled our concluding sentence.

3 Writing and Style Guide

3.1 Write in Complete Sentences

Write any statements you make in complete sentences, rather than in short phrases. In particular, be sure to include:

- Proper use of the technical vocabulary you've learned;
- Proper capitalization (e.g. capitalise the first word in a sentence);
- Punctuation;
- Definite articles ("the," "a," "an," etc.);
- At least one subject, verb, and object; and
- Adverbs as appropriate

On the other hand, don't use abbreviations or symbols (such as @, $\therefore$, $\because$, or =) to replace words in sentences. Some examples follow:

~~$y = 2x - 5$ y-int @ -5~~	should read	The y-intercept of the line $y = 2x - 5$ is $(0, -5)$.
~~$2x = 10$ when x is $= 5$~~	should read	$2x = 10$ when $x = 5$. [or] $2x = 10$ when x is equal to 5.
~~If $x = 5 \therefore 2x = 10$~~	should read	If $x = 5$, then $2x = 10$. [or] If x is 5, then $2x$ is 10.
~~Line $y = 2x$ thru $(-1, -2)$.~~	should read	The line $y = 2x$ passes through the point $(-1, -2)$.

3.2 Declare The (Distinct) Names You Give to Objects

Be sure to declare any new object names you introduce into a problem. This is typically done in one of two ways: either with a "Let..." statement, or with the symbol :=, which can be translated into words as "is defined to be."

For example, in the problem in the "An Example" section, we introduced three new object names: we called the length of the rectangular region by ℓ; the width of the rectangular region by w; and the area of the rectangular region with length ℓ and width w by $A(\ell, w)$. I say that these are *new* object names because the statement of the problem doesn't give us these names; they're names that we made up for ourselves to aid us in our solution, and thus we should inform our readers of what we mean when we refer to ℓ, w, and $A(\ell, w)$, so that they can follow our thinking without having to guess at what we mean.

In particular, we declared $A(\ell, w)$ with the statement:

"Let $A(\ell, w) = \ell w$."

But we could also have declared this function name with the following:

$$A(\ell, w) := \ell w$$

The symbol := is understood to mean that we are *defining* that the function $A(\ell, w)$ is ℓw.

On the other hand, don't reuse object names; use distinct names for distinct (or possibly distinct) objects. For example, if a problem already has one function called $f(x)$ and you need to introduce a second function, give this second function a new name, like $g(x)$, $h(x)$, $f_1(x)$, etc. *Don't* call it $f(x)$, too, otherwise it will be unclear to your reader which function you're referring to when you write the function name $f(x)$.

3.3 Connect Logically Related Equations with $\Rightarrow$

When an equation or a set of equations is used to calculate another equation, we connect these with the symbol $\Rightarrow$, which can be translated into English as the word "implies" or "implies that". This is probably best illustrated with a couple of examples:

$$\begin{aligned} x^2+4x=5 &\Rightarrow x^2+4x-5=0 \\ &\Rightarrow (x-1)(x+5)=0 \\ &\Rightarrow x=1 \text{ or } x=-5 \end{aligned}$$

The first equation above is used to calculate the second, which is used to calculate the third, which in turn leads to the concluding remark: that if the original equation is true, then x must be either 1 or -5 (notice also that we write "or" between these two possibilities in our conclusion above). That is, all of these equations are logically connected: the first implies the second, which implies the third, etc. We indicate this logical relationship between each equation with $\Rightarrow$.

This symbol is especially helpful when a number of ingredients are to be combined to reach a conclusion (as is the case when solving systems of equations—something we will do *quite* a lot of in the middle part of the course). As a simple example:

$$\begin{aligned} x=2 \text{ and } x^2+y^2=4 &\Rightarrow 4+y^2=4 \\ &\Rightarrow y^2=0 \\ &\Rightarrow y=0 \end{aligned}$$

This makes very clear to your reader that you're combining two pieces of information to draw your conclusion—and exactly what those pieces of information are.

Why do we use this symbol? It makes several things crystal clear to a reader: where a logical deduction begins, what ingredients go into that deduction, the conclusion of the deduction, and where the logical deduction ends. It's similar to the reason we write in sentences: to give structure to otherwise scattered symbols in a standardised way.

The $\Rightarrow$ symbol is a special one in math (called a *propositional connective*, or when speaking in the context of formal logic, simply a *connective*), and cannot be substituted for either $\rightarrow$ or $=$. The former is a symbol you've already met before when calculating limits, and it should only be used for this purpose, e.g. $\lim_{h\to 0}(h^2+1) = 1$. On the other hand, $=$ should only be used to connect equal terms in an equation.

3.4 Always Include Variables on Functions

Be sure to include variables on all of your function names, both when introducing a function for the first time, and when talking about or manipulating it. For example, if you wanted to introduce the function $f(x) = x^2 - 3$ into a problem solution, you should write either:

$$\text{“Let } f(x) = x^2 - 3.\text{”} \quad \text{or} \quad \text{“}f(x) := x^2 - 3\text{”}$$

in lieu of

$$\cancel{f := x^2 - 3}$$

There are a couple of reasons for this. First, it makes clear that x is the variable in this function, not just an arbitrary constant. Second, in this course, we will often work with what are called *multivariable* functions, i.e., functions which have more than one variable (similar to $A(\ell, w)$ in the “An Example” section, above). It will be especially important with such functions to point out which letters are variables in such functions, and which aren't.

On a related note, use standard notation to indicate that you're plugging a given value into a function. For example, if you wanted to plug $x = 5$ into the function $f(x) = x^2 - 3$ above, you should write:

$$f(5) = 25 - 3 = 22$$

Rather than, say:

$$\cancel{\text{“At } x = 5, f(x) = 22.\text{”}} \quad \text{or} \quad \cancel{f = 22}$$

3.5 Support Your Claims

You should always strive to give some kind of support for any claim you make, whether that support is a calculation, some proposition or theorem from our text, a definition, or a major, named result. For example, if a claim you make is supported by a major, named result, then say so directly, as we do here:

> Given the function:
>
> $$g(x) = \int_1^x \ln(u)\,\mathrm{d}u,$$
>
> by the second fundamental theorem of calculus, we have that:
>
> $$g'(x) = \ln(x)$$

Here, when we used the second fundamental theorem of calculus to draw the given conclusion, we told the reader as much. One significant reason for doing so is that this result is not frequently used in calculus courses, so it's important to remind the reader that it exists; otherwise, the connection between $g(x)$ and $g'(x)$ might not be obvious to them.

As another example:

> Consider the function $f(x) = x^2$. Recall that, by definition, we have:
>
> $$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{h(2x+h)}{h} \\ &= \lim_{h\to 0} (2x+h) \\ &= 2x \end{aligned}$$

Here, we used the definition of the derivative to calculate the derivative of the function $f(x) = x^2$. Indeed, the first equation *is* the definition of the derivative of this function, so we told the reader as much. Every step after that is due to straightforward algebraic simplification or the evaluation of a straightforward limit, and should be familiar enough to any reader that they need no formal explanation.

3.6 Connect Your Final Answer Back to the Problem Statement

Be sure that your final answer to any given problem connects back to what the problem asked you to find, either through a chain of equations, a standalone equation, or a complete sentence. For example, consider the following problem:

> Give the slope a of the line L that connects the points $(3, 2)$ and $(1, 1)$.

Here is one possible solution:

> *Solution.* By definition, we have:
>
> $$\begin{aligned} a &= \frac{1-2}{1-3} \\ &= \frac{-1}{-2} \\ &= \boxed{\frac{1}{2}} \end{aligned}$$
>
> □

The key thing to note here is that the problem asks us to find a, so our final answer indicates exactly what a is (via a chain of equations): $a = \frac{1}{2}$. This makes it simple for the reader to know exactly what a is meant to be, at a glance. Final answers which fall short include, for example:

$$\cancel{\frac{1}{2}} \quad \text{or} \quad \cancel{m = \frac{1}{2}}$$

The problem asked us to find a. Both answers here are incomplete because they never tell us what a is; neither connects $\frac{1}{2}$ back to a. In particular, the second answer tells us what m is, and while it's true that we often use m to denote the slope of a line, this isn't a universal rule (the letter m doesn't *always* refer to a slope), and besides: the slope of *this* line is called a, so our final answer should talk about a.

Here's a slight tweak to the statement of this problem:

> Give the slope of the line L that connects the points $(3, 2)$ and $(1, 1)$.

The key difference here is that this version of the problem statement doesn't give us a name for the slope of L. Here are a couple different ways to solve this problem:

> *First Solution.* By definition, the slope of L is:
>
> $$\frac{1-2}{1-3} = \frac{-1}{-2} = \frac{1}{2}$$
>
> $\boxed{\text{Therefore, the slope of } L \text{ is } \tfrac{1}{2}.}$ □

In this solution, we answered the question in a complete sentence. The problem asks us to find the slope of L, so our answer says what it is directly: the slope of L is $\frac{1}{2}$.

> *Second Solution.* Let m_L be the slope of L. By definition, we have that:
>
> $$\begin{aligned} m_L &= \frac{1-2}{1-3} \\ &= \frac{-1}{-2} \\ &= \boxed{\frac{1}{2}} \end{aligned}$$
>
> □

In this second solution, we introduce a new variable m_L (though most any variable would be fine (though I would recommend against using x, y, or z); I just chose m_L because we often denote slopes with m, and m_L made it feel to me like there is a direct connection between the slope we're finding and the line L) which is the slope of L, and then tell the reader what m_L is (through a chain of equations). This isn't quite as clean as the previous solution (as the reader would need to look through your work carefully to make sure they know what the symbol m_L is meant to stand for—which could be quite involved in a longer solution), but still: it does make a firm connection between your final answer and what the problem asked us to find.

3.7 Circle Your Final Answer

Always circle your final answer (see the previous subsection for examples of this). This is to remove any doubt about what your final answer is, in case there are a few expressions on the page that might appear to be a final answer.