Optimization

Sketch the following curves using elements of Calculus.

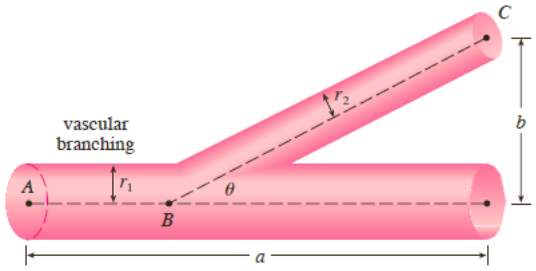
1. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

2. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

3. The blood vascular system consists of blood vessels (arteries, arterioles, capillaries, and veins) that convey blood from the heart to the organs and back to the heart. This system should work so as to minimize the energy expended by the heart in pumping the blood. In particular, the energy is reduced when the resistance of the blood is lowered. One of Poiseuille's Laws gives the resistance R of the blood as

$$R = C\frac{L}{r^4}$$

where L is the length of the blood vessel, r is the radius, and C is a positive constant determined by the viscosity of the blood. The figure shows a main blood vessel with radius r_1 branching at an angle θ into a smaller vessel with radius r_2.



(a) Use Poiseuille's Law to show that the total resistance of the blood along the path ABC is

$$R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

where a and b are the distances shown in the figure.

(b) Prove that this resistance is minimized when

$$\cos\theta = \frac{r_2^4}{r_1^4}$$

(c) Find the optimal branching angle (correct to the nearest degree) when the radius of the smaller blood vessel is two-thirds the radius of the larger vessel.

Antiderivatives

Take the antiderivative.

1. $x + 7$
2. $cosx$
3. π^2
4. $\frac{90}{x^3}$
5. $(x + 2)(3x + 7)$
6. $x(3 - x)^2$
7. $x^{-\frac{1}{3}}$
8. $2\cos x + \sec^2 x$