

Substitution

Sketch the following curves using elements of Calculus.

1. $\int x^3(2 + x^4)^5 dx$

6. $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}} dx$

2. $\int \frac{\sec^2(\frac{1}{x})}{x^2} dx$

7. $\int (x^2 + 1)(x^3 + 3x)^4 dx$

3. $\int (1 - 2x)^9 dx$

8. $\int x(2x + 5)^8 dx$

4. $\int \sec^2\theta d\theta$

9. $\int_0^1 \sqrt[3]{1 + 7x} dx$

5. $\int \sec^2\theta \tan^3\theta d\theta$

10. $\int_{-\pi/4}^{\pi/4} (x^3 + x^4)\tan x dx$

Areas Between Curves

1. $y = (x - 2)^2, y = x$

4. $x = 1 - y^2, x = y^2 - 1$

2. $y = x^2 - 2x, y = x + 4$

5. $y = \frac{1}{4}x^2, y = 2x^2, x + y = 3, x \geq 0$

3. $y = \sqrt{x - 1}, x - y = 1$

6. $y = \frac{1}{x^2}, y = x, y = \frac{1}{8}x$

Challenge Problems

These problems are difficult!

1. If $f(x) = \int_0^x x^2 \sin(t^2) dt$, find $f'(t)$.

2. Find $\frac{d^2}{dx^2} \int_0^x \left(\int_1^{\sin t} \sqrt{1+u^4} du \right) dt$.

3. Evaluate $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$