Limits

Evaluate the following limits:

1. $f(x) = \begin{cases} x+2 & x \leq 6 \\ x^2 - 1 & x > 6 \end{cases}$
   $\lim_{x \to 6} f(x)$

2. $\lim_{h \to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

3. $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

4. $f(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8 - 2x & x < 4 \end{cases}$
   $\lim_{x \to 6} f(x)$

Limits at Infinity

Evaluate the following limits:

1. $\lim_{x \to \infty} \dfrac{4x^2 + 1}{3x^2 + 2x - 1}$

2. $\lim_{x \to -\infty} \dfrac{x^2 + 1}{2x - 1}$

3. $\lim_{x \to \infty} \dfrac{x^3 + 5x^2}{x^8 - 3x^3 + 2x + 1}$

4. $\lim_{x \to \infty} \dfrac{x + 3}{\sqrt{9x^2 - 5x}}$

Limit Definition

Math Logic Symbols
$\forall$: For each / For every
$\exists$: There exists
$\implies$: Implies that / Then / Hence
$\ni$: Such that

Limit Definition

$$\lim_{x \to a} f(x) = L$$

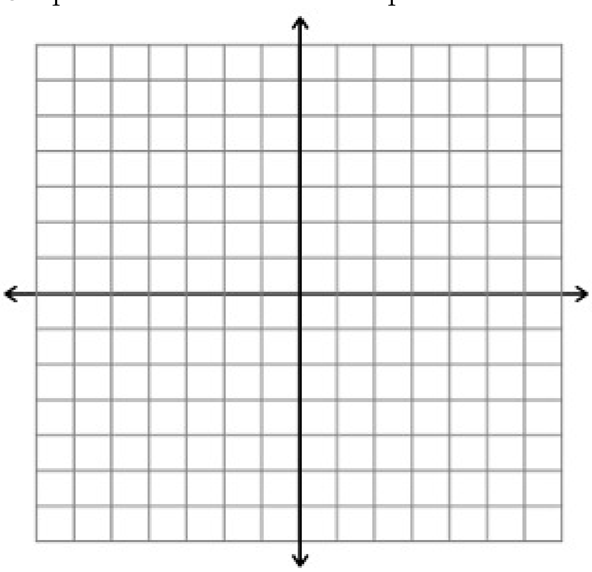
if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that

if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$

Example 1

Let $f(x) = 2x + 3$.

1. Graph the function and the point on the line where x = 2.



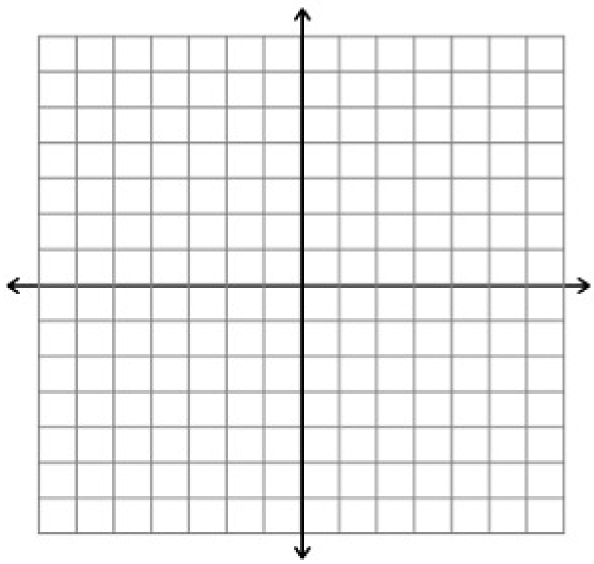
2. We want to prove that $\lim_{x \to 2} f(x) = 7$. If $\varepsilon = 1$, what does δ need to be?

3. Find δ for the limit definition. (δ will be in terms of ε)

Example 2

Let $f(x) = \sqrt{2x}$.

1. Graph the function and the point on the line where x = 0.



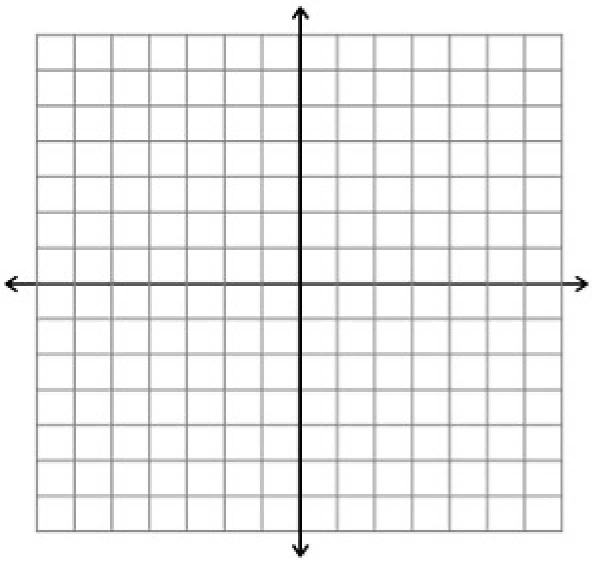
2. We want to prove that $\lim_{x \to 0^+} f(x) = 0$. If $\varepsilon = \frac{1}{2}$, what does δ need to be?

3. Find δ for the limit definition. (δ will be in terms of ε)

Challenge Problems

These are difficult!

1. Draw the graph of the equation $x + |x| = y + |y|$.

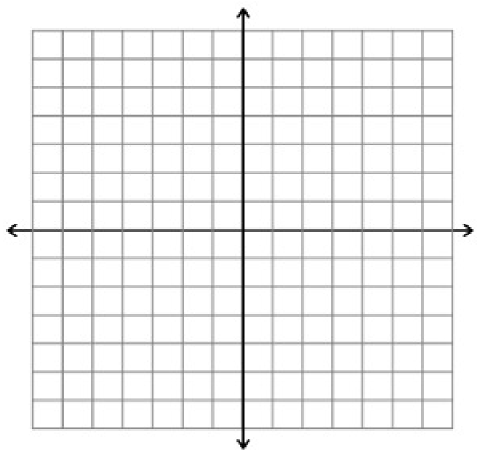


2. Evaluate $\lim_{x \to 0} \frac{|2x-1| - |2x+1|}{x}$

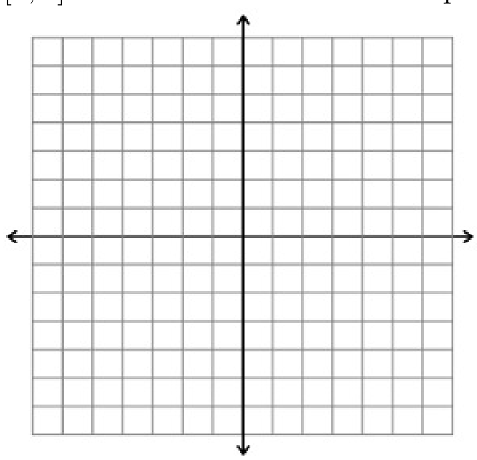
3. Evaluate $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$

4. A **fixed point** is a function f is a number c in its domain such that $f(x) = c$.

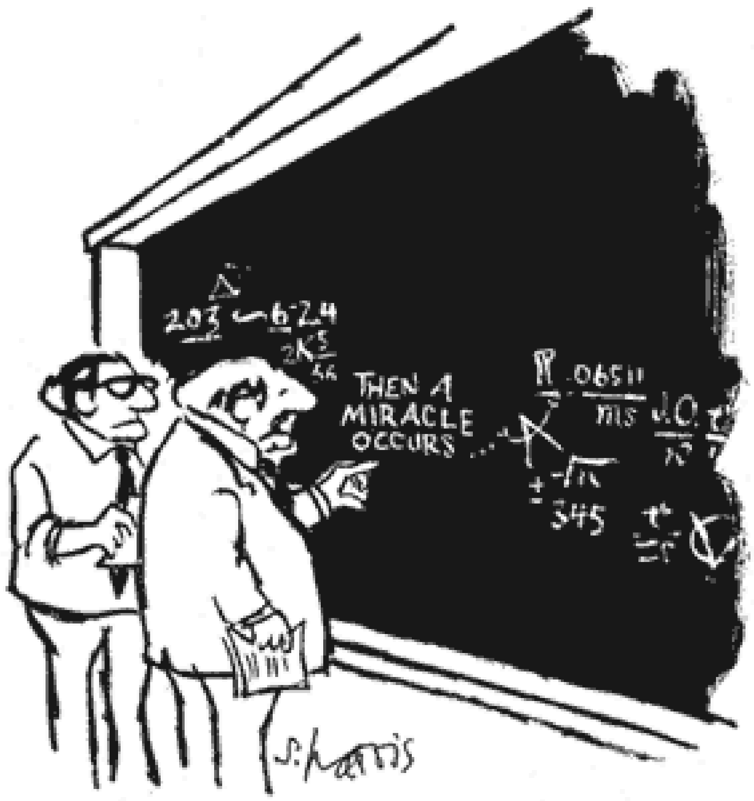
(a) Sketch the graph of a continuous function with domain $[0, 1]$ whose range also lies in $[0, 1]$. Locate a fixed point of f.



(b) Try to draw the graph of a continuous function with domain $[0, 1]$ and range in $[0, 1]$ that does not have a fixed point. What is the obstacle?



(c) Use the Intermediate Value Theorem to prove that any continuous function with domain $[0, 1]$ and range in $[0, 1]$ must have a fixed point.



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."