

Transformations

Vertical Transformations

- $f(x) + c$: Shift up by c .
- $f(x) - c$: Shift down by c .
- $a \cdot f(x)$: Vertical stretch/contraction by a .
- $-f(x)$: Reflection over the x axis.

Horizontal Transformations

- $f(x + c)$: Shift left by c .
- $f(x - c)$: Shift right by c .
- $f(a \cdot x)$: Vertical stretch/contraction by a .
- $f(-x)$: Reflection over the y axis.

Write the formula for the following transformations:

1. $y = \sqrt[4]{x}$ shifted to the right 4 and up 6.

2. $y = \pi^x$ reflected over the x axis, compressed vertically by a factor of 3, and shifted down 2 and to the right 4.

3. $y = x^\pi$ reflected over the y axis, compressed horizontally by a factor of 3, and shifted to the left 6.

4. $y = \frac{\sqrt{x^3-1}}{1+\sqrt[3]{x}}$ stretched horizontally by a factor of 2 and shifted to the left 1.

Compositions

Break the following into a composition of functions:

1. $\sqrt[3]{1+4x}$

5. $\sin(\sin(\sin(x)))$

2. $\tan \pi x$

6. $\sqrt{x + \sqrt{x + \sqrt{x}}}$

3. $\sin(\tan 2x)$

7. $[x + (x + \sin^2 x)^3]^4$

4. $\frac{1}{\sqrt[3]{x^2+x+1}}$

8. $\cos \sqrt{\sin(\tan \pi x)}$

Trigonometric Functions

Draw the 30-60-90 triangle below:

Evaluate the following:

1. $\sin\left(\frac{\pi}{3}\right)$

3. $\cos\left(\frac{\pi}{3}\right)$

5. $\tan\left(\frac{\pi}{3}\right)$

2. $\sin\left(\frac{\pi}{6}\right)$

4. $\cos\left(\frac{\pi}{6}\right)$

6. $\tan\left(\frac{\pi}{6}\right)$

Draw the 45-45-90 triangle below:

Evaluate the following:

1. $\sin\left(\frac{\pi}{4}\right)$

2. $\cos\left(\frac{\pi}{4}\right)$

3. $\tan\left(\frac{\pi}{4}\right)$

Important Trig Identities

- $-1 \leq \sin x \leq 1$
- $-1 \leq \cos x \leq 1$
- $\tan x = \frac{\sin x}{\cos x}$
- $\sec x = \frac{1}{\cos x}$
- $\csc x = \frac{1}{\sin x}$
- $\cot x = \frac{1}{\tan x}$
- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$

Can you get $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$ from $\sin^2 x + \cos^2 x = 1$?

Can we think of $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$ in terms of transformations? [Hint: Think of the graph of $\sin x$ and $\cos x$.]