Max/Min

Find the critical numbers.

1. $f(x) = 2x^3 - 3x^2 - 36x$
2. $g(x) = \frac{x-1}{x^2-x+1}$
3. $f(x) = 2\cos\theta + \sin^2\theta$
4. $f(x) = \sqrt{1-x^2}$

Find the absoluate max and min on the given intervals.

1. $f(x) = 12 + 4x - x^2$, $[0, 5]$
2. $f(x) = (x^2 - 1)^3$, $[-1, 2]$
3. $f(x) = x\sqrt[3]{4 - x^2}$
4. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, $[-2, 3]$

Mean Value Theorem

Rolle's Theorem

Let f be a function that satisfies the following:

1. f is continuous on $[a, b]$.
2. f is differentiable on (a, b).
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

Mean Value Theorem

Let f be a function that satisfies the following:

1. f is continuous on $[a, b]$.
2. f is differentiable on (a, b).

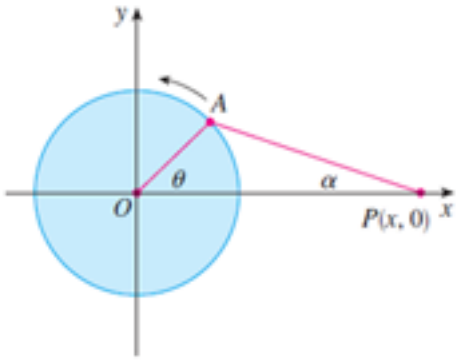
Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed.
Hint: Look at $f(t) = g(t) - h(t)$ where g and h are the position functions of the two runners.

Homework Problems

11. The figure shows a rotating wheel with radius 40 cm and a connecting rod AP with length 1.2 m. The pin P slides back and forth along the x-axis as the wheel rotates counterclockwise at a rate of 360 revolutions per minute.



(a) Find the angular velocity of the connecting rod, $\frac{d\alpha}{dt}$, in radians per second, when $\theta = \frac{\pi}{3}$.

(b) Express the distance $x = |OP|$ in terms of θ.

(c) Find an expression for the velocity of the pin P in terms of θ.

(16) Let $P(x_1, y_1)$ be a point on the parabola $y^2 = 4px$ with focus $F(p, 0)$. Let α be the angle between the parabola and the line segment FP, and let β be the angle between the horizontal line $y = y_1$ and the parabola as in the figure (Stewart pg. 196). Prove that $\alpha = \beta$.

Hint: Use the fact that if lines L_1 and L_2 intersect at an angle θ, then $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

(17) In the figure (Stewart pg. 196), C is a semicircle with center O. A ray of light coming in toward the mirror parallel to the axis along the line PQ will be reflected to the point R on the axis so that $\angle PQO = \angle OQR$. What happens to the point R as P is taken closer and closer to the axis?