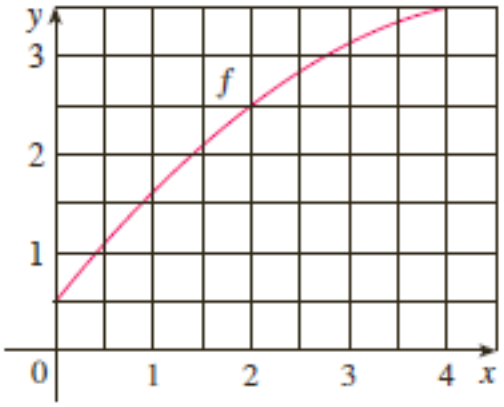
Approximate Integration

- Let $I = \int_0^4 f(x)\,dx$, where f is the function whose graph is shown.



1. Use the graph to find L_2, R_2, and M_2.

2. Are these underestimates or overestimates of I?

3. Use the graph to find T_2. How does it compare with I?

4. For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order.

- Estimate $\int_0^1 \cos(x^2)\ dx$, using (a) the Trapezoidal Rule and (b) the Midpoint Rule, each with $n = 4$. From a graph of the integrand, decide whether your answers are underestimates or overestimates. What can you conclude about the true value of the integral?

- Show that if f is a polynomial of degree 3 or lower, then Simpson's Rule gives the exact value of $\int_a^b f(x)\ dx$.

Techniques of Integration

- $\int_1^2 \frac{(x+1)^2}{x}\ dx$

- $\int e^x \cos x\ dx$

Improper Integrals

- Explain why each of the following integrals is improper.

1. $\displaystyle\int_1^2 \frac{x}{x-1}\,dx$

2. $\displaystyle\int_0^\infty \frac{1}{1+x^3}\,dx$

3. $\displaystyle\int_{-\infty}^\infty x^2 e^{-x^2}\,dx$

4. $\displaystyle\int_0^{\frac{\pi}{4}} \cot x\,dx$

- Determine if the following integrals are convergent or divergent. If the integral is convergent, evaluate it.

  1. $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\ dx$

  2. $\displaystyle\int_{-2}^{3} \frac{1}{x^4}\ dx$

- 1. Show that $\displaystyle\int_{-\infty}^{\infty} x\ dx$ is divergent.

  2. Show that $\displaystyle\lim_{t\to\infty}\int_{-t}^{t} x\ dx = 0$.

     This shows that we can't define $\displaystyle\int_{-\infty}^{\infty} f(x)\ dx = \lim_{t\to\infty}\int_{-t}^{t} f(x)\ dx$