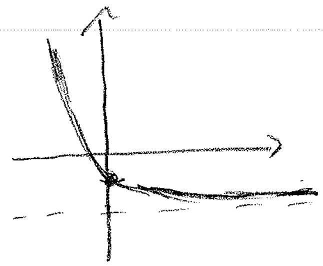
6.2 # 8, 14, 18, 28, 32, 36, 42, 44, 52, 84, 86

8) Make a rough sketch of the function. Do not use a calculator.

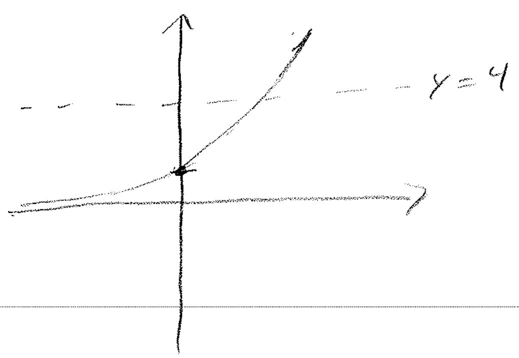
$$y = (.5)^x - 2$$



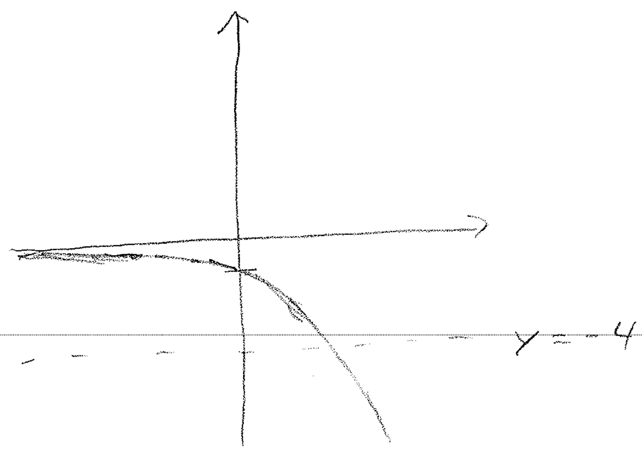
14) Starting with the graph of $y = e^x$, find the equation of the graph that results from

a) reflecting about the line $y = 4$

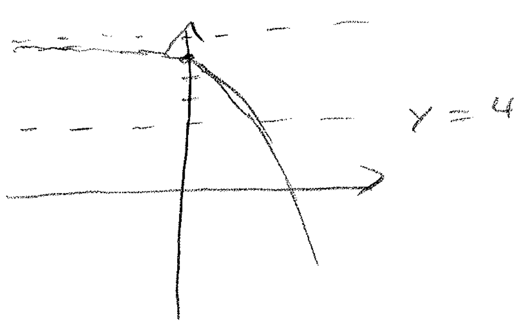
e^x



$-e^x$



$-e^x + 8$

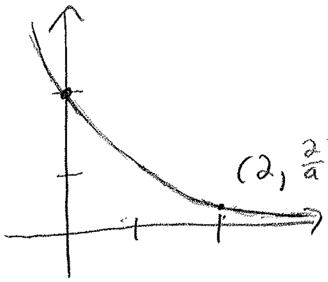


$$\boxed{y = -e^x + 8}$$

b) reflecting about the line $x=2$

Similarly, $y = e^{-(x-4)}$

18) Find the exponential function $f(x) = Ca^x$ whose graph is given.



$\underline{(0,2)}$ $\quad 2 = Ca^0 = C$

$\underline{(2, \frac{2}{9})}$ $\quad \frac{2}{9} = 2a^2$

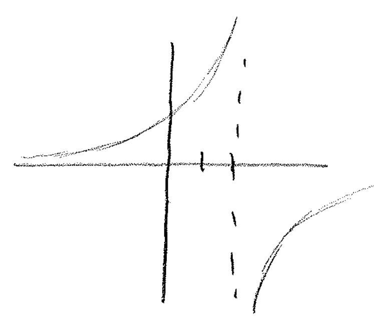
$\frac{1}{9} = a^2$

$a = \pm \frac{1}{3}$

Since $a > 0$, $a = \frac{1}{3}$

$$\boxed{f(x) = 2\left(\tfrac{1}{3}\right)^x}$$

$\frac{3}{2-x}$



28) Find the limit.

$$\lim_{x \to 2^-} e^{\frac{3}{2-x}}$$

$$\lim_{x \to 2^-} \frac{3}{2-x} = \infty$$

Let $u = \frac{3}{2-x}$

$$\lim_{x \to 2^-} e^{\frac{3}{2-x}} = \lim_{u \to \infty} e^u = \infty$$

Differentiate the function

32) $k(r) = e^r + r^e$

$$\boxed{k'(r) = e^r + e r^{e-1}}$$

36) $y = e^{-2t} \cos 4t$

$$\boxed{y' = -2e^{-2t} \cos 4t - 4e^{-2t} \sin 4t}$$

42) $y = x^2 e^{-\frac{1}{x}}$

$$y' = 2xe^{-\frac{1}{x}} + x^2 e^{-\frac{1}{x}} (-x^{-1})'$$

$$= 2xe^{-\frac{1}{x}} + x^2 e^{-\frac{1}{x}} (x^{-2})$$

$$= \boxed{2xe^{-\frac{1}{x}} + e^{-\frac{1}{x}}}$$

44) $y = e^{k \tan \sqrt{x}}$

$$y' = e^{k \tan \sqrt{x}} (\sec^2 \sqrt{x}) \left(\frac{1}{2\sqrt{x}}\right)$$

$$= \boxed{\frac{\sec^2 \sqrt{x} \; e^{k \tan \sqrt{x}}}{2\sqrt{x}}}$$

52) Find the tangent line to curve at the given point.

$y = \frac{e^x}{x}$, $(1, e)$

$$y' = \frac{e^x}{x} + e^x\left(-\frac{1}{x^2}\right)$$

$$y'(1) = e - e = 0$$

$$y - e = 0(x - 1)$$

$$\boxed{y = e}$$

Here, I used the product rule for $e^x (x^{-1})$. You may also use the quotient rule for $\frac{e^x}{x}$.

Evaluate the integral.

84) $\int \frac{(1+e^x)^2}{e^x}\,dx$

$= \int \frac{1+2e^x+e^{2x}}{e^x}\,dx$

$= \int e^{-x}+2+e^x\,dx$

$= \boxed{-e^{-x}+2x+e^x+C}$

86) $\int e^x(4+e^x)^5\,dx$ $\quad\begin{vmatrix} u=4+e^x \\ du=e^x dx\end{vmatrix}$

$= \int u^5\,du$

$= \frac{1}{6}u^6+C$

$= \boxed{\frac{1}{6}(4+e^x)^6+C}$

6.2) #54, 57, 94 6.3) 6, 8, 12, 26, 28, 34, 40, 52, 54, 56, 64

54) Find an equation of the tangent line to the curve $xe^y+ye^x=1$ at the point $(0,1)$.

- $e^y+xe^y y'+y'e^x+ye^x=0$

  $xe^y y'+y'e^x=-e^y-ye^x$

  $y'(xe^y+e^x)=-e^y-ye^x$

  $y'=\frac{-e^y-ye^x}{xe^y+e^x}$

- At $(0,1)$: $y'=\frac{-e^1-1e^0}{0e^1+e^0}=-e-1$

- $y-1=(-e-1)(x-0)$

  $\boxed{y=(-e-1)x+1}$

57) For what values of r does the function $y = e^{rx}$ satisfy the differential equation $y'' + 2y' + y = 0$

$$y = e^{rx}$$
$$y' = re^{rx}$$
$$y'' = r^2 e^{rx}$$

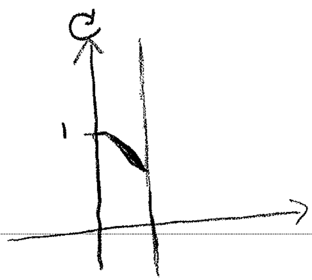
$$r^2 e^{rx} + 6re^{rx} + 8e^{rx} = 0$$

$$e^{rx}(r^2 + 6r + 8) = 0$$

$e^{rx} \neq 0$, so $r^2 + 6r + 8 = 0$

$$(r+4)(r+2) = 0$$
$$\boxed{r = -2, -4}$$

94) Find the volume of the solid obtained by rotating about the y-axis the region bounded by the curves $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$



We want our bounds in terms of x and we are rotating around the y-axis, so we should use cylindrical shells.

$$V = \int_0^1 2\pi x e^{-x^2}\,dx$$

$u = -x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$$= -\pi \int_0^{-1} e^u\,du$$

$$= -\pi \left[e^u\right]_0^{-1} = -\pi(e^{-1} - e^0)$$
$$= -\pi(e^{-1} - 1)$$
$$= \boxed{\pi\left(1 - \frac{1}{e}\right)}$$

6) Find the exact value of

a) $\log_{1.5} 2.25$

$= \log_{\frac{3}{2}} \frac{9}{4} = \log_{\frac{3}{2}} \left(\frac{3}{2}\right)^2$

$= \boxed{2}$

b) $\log_5 4 - \log_5 500$

$= \log_5 \frac{4}{500} = \log_5 125 = \boxed{3}$

8) Find the exact value of

a) $e^{-2\ln 5}$

$= e^{\ln 5^{-2}}$

$= 5^{-2} = \boxed{\frac{1}{25}}$

b) $\ln\left(\ln\left(e^{e^{10}}\right)\right)$

$= \ln\left(e^{10}\right)$

$= \boxed{10}$

12) Use the properties of logarithms to expand the quantity.

$\ln\left(s^4\sqrt{t\sqrt{u}}\right)$

$= \ln(s^4) + \ln\left(\sqrt{t\sqrt{u}}\right)$

$= 4\ln s + \ln\left(\left(t\sqrt{u}\right)^{\frac{1}{2}}\right)$

$= 4\ln s + \frac{1}{2}\ln\left(t\sqrt{u}\right)$

$= 4\ln s + \frac{1}{2}\left(\ln t + \ln\sqrt{u}\right)$

$= 4\ln s + \frac{1}{2}\ln t + \frac{1}{2}\ln u^{\frac{1}{2}}$

$= \boxed{4\ln s + \frac{1}{2}\ln t + \frac{1}{4}\ln u}$

26) $f(x) = \ln(x-1) - 1$

a) What are the domain and range of f?

Domain : $x - 1 > 0$

$\boxed{x > 1 \quad \text{or} \quad (1, \infty)}$

Range : $\boxed{(-\infty, \infty) \quad \text{or All real numbers or} \quad \mathbb{R}}$

b) What is the x-intercept of the graph of f?

$0 = \ln(x-1) - 1$

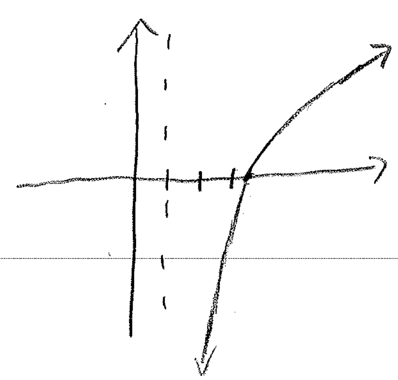
$1 = \ln(x-1)$

$e^1 = e^{\ln(x-1)}$

$e = x - 1$

$\boxed{x = e + 1}$

c) Sketch the graph of f.



Solve each equation for x.

28) a) $\ln(x^2-1)=3$

$$e^{\ln(x^2-1)}=e^3$$

$$x^2-1=e^3$$

$$x^2=e^3+1$$

$$\boxed{x=\pm\sqrt{e^3+1}}$$

b) $e^{2x}-3e^x+2=0$

Let $u=e^x$

$$u^2-3u+2=0$$

$$(u-2)(u-1)=0$$

$$u=1,2$$

$e^x=1$ | $e^x=2$

$\ln e^x=\ln 1$ | $\ln e^x=\ln 2$

$\boxed{x=0}$ | $\boxed{x=\ln 2}$

34) $e^{e^x}=10$

$$\ln e^{e^x}=\ln 10$$

$$e^x=\ln 10$$

$$\ln e^x=\ln(\ln 10)$$

$$\boxed{x=\ln(\ln 10)}$$

40) Solve each inequality for x

a) $1<e^{3x-1}<2$

$\ln x$ is increasing $\rightarrow$ $\ln 1<\ln e^{3x-1}<\ln 2$

$$0<3x-1<\ln 2$$

$$1<3x<1+\ln 2$$

$$\boxed{\frac{1}{3}<x<\frac{1+\ln 2}{3}}$$

b) $1-2\ln x<3$

$$-2\ln x<2$$

$$\ln x>-1$$

$e^{\ln x}>e^{-1}$ $\leftarrow$ e^x is increasing

$$\boxed{x>\frac{1}{e}}$$

52) Find the limit.

$$\lim_{x \to \infty} [\ln(2+x) - \ln(1+x)]$$

$$= \lim_{x \to \infty} \left[\ln\left(\frac{2+x}{1+x}\right)\right]$$

$$\lim_{x \to \infty} \frac{2+x}{1+x} = 1$$

Let $u = \frac{2+x}{1+x}$

$$= \lim_{u \to 1} \ln(u)$$

$$= \ln(1) = \boxed{0}$$

54) Find the domain of the function.

$$f(x) = \ln x + \ln(2-x)$$

$x > 0$ and $2 - x > 0$

$x > 0$ and $2 > x$

$\boxed{0 < x < 2 \text{ or } (0,2)}$

56) $f(x) = \ln(2 + \ln x)$

a) Find the domain of f.

$x > 0$ and $2 + \ln x > 0$

$\ln x > -2$

$e^{\ln x} > e^{-2}$

$x > e^{-2}$

$\boxed{x > \frac{1}{e^2} \text{ or } \left(\frac{1}{e^2}, \infty\right)}$

b) Find f^{-1} and its domain.

$$y = \ln(2 + \ln x)$$

$$e^y = e^{\ln(2+\ln x)}$$

$$e^y = 2 + \ln x$$

$$e^y - 2 = \ln x$$

$$e^{\ln x} = e^{e^y - 2}$$

$$x = e^{e^y - 2}$$

$$\boxed{f^{-1}(x) = e^{e^y - 2}}$$

$\boxed{\text{Domain: } (-\infty, \infty) \text{ or all real numbers or } \mathbb{R}}$

64) Find the inverse function of

$$y = \frac{e^x}{1+2e^x}$$

$$y + 2ye^x = e^x$$

$$y = e^x - 2ye^x$$

$$y = e^x(1-2y)$$

$$e^x = \frac{y}{1-2y}$$

$$\ln e^x = \ln\left(\frac{y}{1-2y}\right)$$

$$x = \ln\left(\frac{y}{1-2y}\right)$$

$$\boxed{f^{-1}(x) = \ln\left(\frac{x}{1-2x}\right)}$$

6.4) #4, 6, 8, 12, 20, 30, 32, 36, 52, 76, 80

Differentiate the function.

4) $f(x) = \ln(\sin^2 x)$

$$f'(x) = \frac{1}{\sin^2 x}(2\sin x)(\cos x)$$

$$= 2\,\frac{\cos x}{\sin x} = \boxed{2\cot x}$$

6) $y = \dfrac{1}{\ln x} = (\ln x)^{-1}$

$$y' = -(\ln x)^{-2}\left(\frac{1}{x}\right)$$

$$= \boxed{-\frac{1}{x(\ln x)^2}}$$

8) $f(x) = \log_5 (x e^x)$

$$f'(x) = \frac{1}{xe^x \ln 5}\left(e^x + xe^x\right)$$

$$= \frac{e^x + xe^x}{xe^x \ln 5}$$

$$= \frac{e^x(1+x)}{xe^x \ln 5} = \boxed{\frac{1+x}{x\ln 5}}$$

12) $h(x) = \ln\left(x + \sqrt{x^2-1}\right)$

$$h'(x) = \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)\right)$$

$$= \frac{1 + \frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}$$

$$= \boxed{\frac{\sqrt{x^2-1} + x}{x\sqrt{x^2-1} + x^2 - 1}}$$

20) $H(z) = \ln\sqrt{\frac{a^2 - z^2}{a^2+z^2}} = \ln \frac{\sqrt{a^2-z^2}}{\sqrt{a^2+z^2}}$

$$= \ln\sqrt{a^2-z^2} - \ln\sqrt{a^2+z^2}$$

$$= \frac{1}{2}\ln(a^2-z^2) - \frac{1}{2}(a^2+z^2)$$

$$H'(z) = \frac{1}{2}\left(\frac{1}{a^2-z^2}\right)(-2z) - \frac{1}{2}\left(\frac{1}{a^2+z^2}\right)(2z)$$

$$= -\frac{z}{a^2-z^2} - \frac{z}{a^2+z^2}$$ ← this answer is fine

$$= \frac{-za^2 - z^3 - za^2 + z^3}{(a^2-z^2)(a^2+z^2)} = \boxed{\frac{-2za^2}{(a^2-z^2)(a^2+z^2)}}$$

30) Find y' and y''.

$y = \ln(\sec x + \tan x)$

$$y' = \frac{1}{\sec x + \tan x}\left[\sec x \tan x + \sec^2 x\right]$$

$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \boxed{\sec x = y'}$$

$$\boxed{y'' = \sec x \tan x}$$

32) Differentiate f and find the domain of f.

$f(x) = \sqrt{2 + \ln x}$

$f'(x) = \frac{1}{2}(2 + \ln x)^{-\frac{1}{2}}\left(\frac{1}{x}\right)$

$= \boxed{\dfrac{1}{2x\sqrt{2 + \ln x}}}$

Domain: $x > 0$

$2 + \ln x \geq 0$

$\ln x \geq -2$

$\boxed{x \geq e^{-2}}$ e^x is increasing.

$\boxed{[e^{-2}, \infty)}$

36) If $f(x) = \ln(1 + e^{2x})$, find $f'(0)$.

$f'(x) = \dfrac{1}{1 + e^{2x}}\left(2e^{2x}\right) = \dfrac{2e^{2x}}{1 + e^{2x}}$

$f'(0) = \dfrac{2e^0}{1 + e^0} = \dfrac{2}{1+1} = \boxed{1}$

52) Use logarithmic differentiation to find the derivative of

$y = (\sin x)^{\ln x}$

$\ln y = \ln(\sin x)^{\ln x}$

$\ln y = \ln x \ln(\sin x)$

$\frac{1}{y} y' = \dfrac{\ln(\sin x)}{x} + \dfrac{x \cos x}{\sin x}$

$y' = y\left(\dfrac{\ln(\sin x)}{x} + \dfrac{x \cos x}{\sin x}\right)$

$\boxed{y' = (\sin x)^{\ln x}\left[\dfrac{\ln(\sin x)}{x} + x \cot x\right]}$

Evaluate the integral.

76) $\int \frac{\sin(\ln x)}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}\, dx$

$= \int \sin u \, du$

$= -\cos u + C$

$= \boxed{-\cos(\ln x) + C}$

80) $\int \frac{e^x}{e^x + 1}\, dx$ $\quad u = e^x + 1$, $du = e^x dx$

$= \int \frac{1}{u}\, du$

$= \ln|u| + C$

$= \ln|e^x + 1| + C$ $\quad$ since $e^x + 1 > 0$

$= \boxed{\ln(e^x + 1) + C}$

6.6 # 5(b), 7, 24, 28, 36, 39, 44, 58, 60, 61, 65, 68

Find the exact value of each expression.

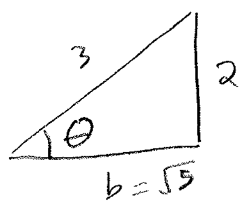
5b) $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right)$

$\sin\left(\frac{7\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So, $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right) = \boxed{\frac{\pi}{3}}$

7) $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

$\sin\theta = \frac{opp}{hyp} = \frac{2}{3}$



$b^2 + 2^2 = 3^2$

$b^2 = 9 - 4$

$b = \sqrt{5}$

$\tan\theta = \boxed{\frac{2}{\sqrt{5}} \text{ OR } = \frac{2\sqrt{5}}{5}}$

Find the derivative.

24) $y = \cos^{-1}(\sin^{-1} t)$

$y' = \boxed{-\frac{1}{\sqrt{1-(\sin^{-1}t)^2}}\left[\frac{1}{\sqrt{1-t^2}}\right]}$

28) $F(\theta) = \arcsin\sqrt{\sin\theta}$

$F'(\theta) = \frac{1}{\sqrt{1-(\sqrt{\sin\theta})^2}}\left(\frac{1}{2}(\sin\theta)^{-\frac{1}{2}}\right)(\cos\theta)$

$= \frac{\cos\theta}{2\sqrt{1-\sin\theta}\sqrt{\sin\theta}} = \boxed{\frac{\cos\theta}{2\sqrt{\sin\theta - \sin^2\theta}}}$

36) Find the derivative of the function. Find the domains of the function and its derivative.

$f(x) = \arcsin(e^x)$

$$f'(x) = \frac{1}{\sqrt{1-e^{2x}}}(e^x)$$

Domain of f: $-1 \le e^x \le 1$

$0 < e^x \le 1$

$\boxed{-\infty < x \le 0 \quad \text{or} \quad (-\infty, 0]}$

Domain of f': $1 - e^{2x} > 0$

$e^{2x} < 1$

$2x < \ln 1$

$2x < 0$

$\boxed{x < 0 \quad \text{or} \quad (-\infty, 0)}$

39) If $g(x) = x\sin^{-1}\left(\frac{x}{4}\right) + \sqrt{16-x^2}$, find $g'(2)$.

$$g'(x) = \sin^{-1}\left(\frac{x}{4}\right) + \frac{x}{4\sqrt{1-\left(\frac{x}{4}\right)^2}} + \frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \sin^{-1}\left(\frac{x}{4}\right) + \frac{x}{\sqrt{16-x^2}} - \frac{x}{\sqrt{16-x^2}}$$

$$= \sin^{-1}\left(\frac{x}{4}\right)$$

$$g'(2) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$$

44) Find the limit.

$$\lim_{x\to\infty} \arccos\left(\frac{1+x^2}{1+2x^2}\right)$$

$$\lim_{x\to\infty} \frac{1+x^2}{1+2x^2} = \frac{1}{2}$$

Let $u = \dfrac{1+x^2}{1+2x^2}$

$$\lim_{u\to\frac{1}{2}} \arccos(u) = \arccos\left(\tfrac{1}{2}\right)$$

$$= \boxed{\frac{\pi}{3}}$$

58) Find $f(x)$ if $f'(x) = \dfrac{4}{\sqrt{1-x^2}}$ and $f\left(\tfrac{1}{2}\right) = 1$

$$f(x) = \int \frac{4}{\sqrt{1-x^2}}\,dx = 4\arcsin x + C$$

$$4\arcsin\left(\tfrac{1}{2}\right) + C = 1$$

$$\frac{2\pi}{3} + C = 1$$

$$C = 1 - \frac{2\pi}{3}$$

$$\boxed{f(x) = 4\arcsin x + 1 - \frac{2\pi}{3}}$$

Evaluate the integral.

60) $\displaystyle\int_{1/2}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}} = 4\arcsin x \Big|_{1/2}^{\frac{1}{\sqrt{2}}} = 4\arcsin\left(\tfrac{\sqrt{2}}{2}\right) - 4\arcsin\left(\tfrac{1}{2}\right)$

$$= \frac{4\pi}{4} - \frac{4\pi}{6} = \boxed{\frac{\pi}{3}}$$

61) $\displaystyle\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ $\qquad u = \sin^{-1}x$, $\quad du = \dfrac{1}{\sqrt{1-x^2}}\,dx$

$$= \int_0^{\frac{\pi}{6}} u\,du = \frac{1}{2}u^2\Big|_0^{\frac{\pi}{6}} = \boxed{\frac{\pi^2}{72}}$$

65) $\int \frac{dx}{\sqrt{1-x^2}\sin^{-1}x}$

$u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|\sin^{-1}x| + C}$

68) $= \int \frac{x}{1+x^4}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$

$= \frac{1}{2}\arctan u + C$

$= \boxed{\frac{1}{2}\arctan x^2 + C}$