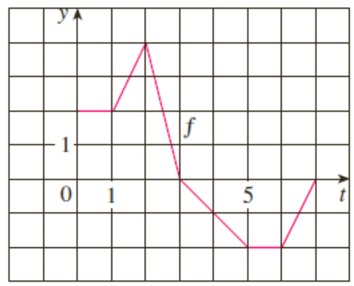
Integrals

- Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.



1. Evaluate $g(0), g(1), g(2), g(3), g(6)$.

2. On what interval is g increasing?

3. Where does g have a maximum value?

4. Sketch a rough graph of g.

- Use the Fundamental Theorem of Calculus to find the derivative of the function.

1. $g(s) = \int_5^s (t - t^2)^8 \, dt$
2. $F(x) = \int_x^\pi \sqrt{1 + \sec t} \, dt$
3. $h(x) = \int_2^{\frac{1}{x}} \sin^4 t \, dt$
4. $y = \int_0^{\tan x} \sqrt{t + \sqrt{t}} \, dt$

- Evaluate the integral.

1. $\int_1^9 \sqrt{x} \, dx$
2. $\int_{\frac{\pi}{6}}^\pi \sin\theta \, d\theta$
3. $\int_0^1 (u+2)(u-3) \, du$
4. $\int_1^2 \frac{v^5 + 3v^6}{v^4} \, dv$

- What is wrong with the equation?

$$\int_{-2}^{1} x^{-4}\, dx = \frac{x^{-3}}{-3}\Big|_{-2}^{1} = -\frac{3}{8}$$

- The **sine integral function**,

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that the limit is 1 when $t \to 0$. So, we define $f(0) = 1$ and this makes f a continuous function everywhere.]

1. At what values of x does this function have local maximum values?

2. Find the coordinates of the first inflection point to the right of the origin.

- Evaluate the limit by first recognizing the sum as a Reimann sum for a function defined on $[0, 1]$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$$

- Find the derivative of the function.

$$h(x) = \int_{\sqrt{x}}^{x^3} \cos(t^2)\ dt$$

- If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t)\ dt.$$

- Evaluate the integral.

$$\int_2^5 |x - 3|\ dx$$