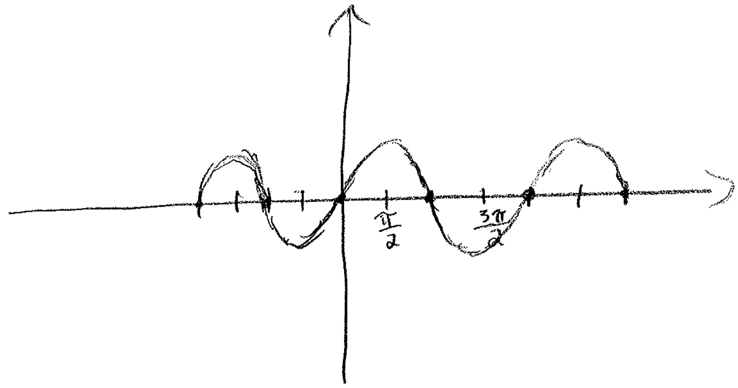
$\sin x$



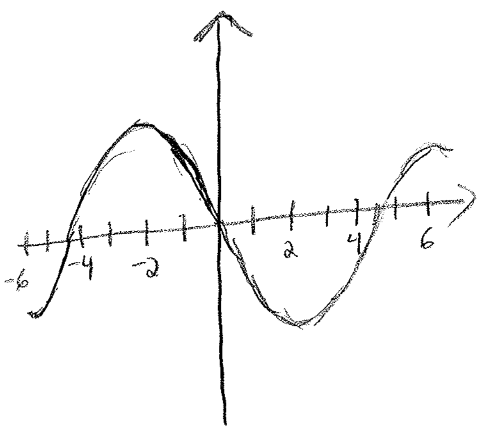
Thus we get local maxima for $x = \pi, -2\pi, 3\pi, -4\pi, \ldots$

(2) Find the coordinates of the first inflection point to the right of the origin.

Inflection points occur when $Si''(x) = 0$

$$S''(x) = \left(\frac{\sin x}{x}\right)' = \frac{\cos x}{x} - \frac{\sin x}{x^2}$$

If we graph $\frac{\cos x}{x} - \frac{\sin x}{x^2}$, it will look something like:



← We can graph this using Wolfram Alpha.

Wolfram Alpha (or any other Math software) tells us that

$\frac{\cos x}{x} - \frac{\sin x}{x^2}$ has a root at about 4.4934.

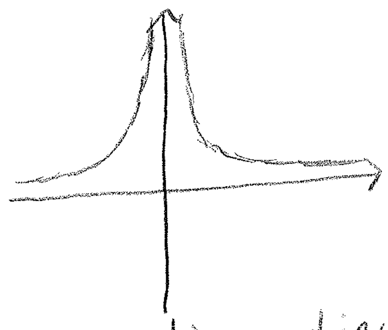
$Si(4.4934) \approx 1.6556$.

So, there is an inflection point at $(4.4934, 1.6556)$.

- What is wrong with the equation?

$$\int_{-2}^{1} x^{-4}\,dx = \frac{x^{-3}}{-3}\Big|_{-2}^{1} = -\frac{3}{8}$$

Graph of x^{-4}:



This graph has a discontinuity at $x=0$, so we cannot integrate it from -2 to 1.

- The <u>sine integral function</u> is

$$Si(x) = \int_0^x \frac{\sin t}{t}\,dt.$$

① At what values of x does this function have local maximum values?

$Si(x)$ has local maximums when $Si'(x)$ goes from positive to negative.

From the Fundamental Theorem of Calculus,

$$Si'(x) = \frac{\sin x}{x}.$$

For local extrema, $\frac{\sin x}{x} = 0$, so $\sin x = 0$, $x \neq 0$.

These occur at $x = n\pi$, $n = 1, -1, 2, -2, 3, -3, \ldots$

Since $\frac{\sin x}{x}$ will keep the sign of sin for $x>0$, but change the sign of sin for $x<0$, we will look for when $\sin x$ goes from positive to negative for $x>0$. However, we will look for when $\sin x$ goes from negative to positive for $x<0$.

- Evaluate the limit by first recognizing the sum as a Reimann sum for a function defined on $[0,1]$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i^3}{n^3}\right) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3$$

$$\Delta x = \frac{1}{n} \to b-a=1$$

$$x_i = \frac{i}{n} \to a=0$$

So, $a=0$, $b=1$

$$\int_0^1 x^3\,dx = \frac{1}{4}x^4 \Big|_0^1 = \frac{1}{4}(1)^4 - 0 = \frac{1}{4}$$

- If f and g are differentiable functions, find a formula for

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t)\,dt$$

$$= \frac{d}{dx} \int_{g(x)}^{0} f(t)\,dt + \frac{d}{dx} \int_{0}^{h(x)} f(t)\,dt$$

$$= -\frac{d}{dx} \int_{0}^{g(x)} f(t)\,dt + \frac{d}{dt} \int_{0}^{h(x)} f(t)\,dt$$

$$= -f(g(x))\,g'(x) + f(h(x))\,h'(x)$$

- Evaluate the integral.

$$\int_2^5 |x-3|\,dx$$

$$|x-3| = \begin{cases} x-3 & \text{if } x>3 \\ 3-x & \text{if } x<3 \end{cases}$$

$$\int_2^5 |x-3|\,dx = \int_2^3 (3-x)\,dx + \int_3^5 (x-3)\,dx$$

$$= 3x - \frac{x^2}{2}\Big|_2^3 + \frac{1}{2}x^2 - 3x\Big|_3^5$$

$$= \left(3(3) - \frac{3^2}{2} - 3(2) + \frac{2^2}{2}\right) +$$

$$\left(\frac{1}{2}(5)^2 - 3(5) - \frac{1}{2}(3)^2 + 3(3)\right)$$

$$= \left(9 - \frac{9}{2} - 6 + 2\right) + \left(\frac{25}{2} - 15 - \frac{9}{2} + 9\right)$$

$$= 5 - \frac{9}{2} + 2 = \frac{5}{2}$$