Additional Problems

HW #6

March 30, 2016

Problem 1. Show that

$$10^{\log_{10}(x)} = 10^{\frac{\ln x}{\ln 10}} \tag{1}$$

Notice that after taking the logarithm with base 10 on both sides, we get the "change of base" formula (of course, using the change of base formula to prove this would be missing the point.)

Proof. The left side equals x because $\log_{10} x$ is the inverse function of 10^x.

To show that the right hand side equals x,

$$\begin{aligned} 10^{\frac{\ln x}{\ln 10}} &= (e^{\ln 10})^{\frac{\ln x}{\ln 10}} && (2)\\ &= e^{\ln 10 \frac{\ln x}{\ln 10}} && (3)\\ &= e^{\ln x} && (4)\\ &= x && (5) \end{aligned}$$

□

Note. There are more than one proof to this problem and some of you showed other correct proofs in the homework. However, if you use the fact that

$$\log_{10}(x) = \frac{\ln x}{\ln 10} \tag{6}$$

You would be correct since the problem didn't forbid you from doing so. However that in a way destroys the purpose of this problem: we are trying to prove the change of base formula without assuming it to be true in the first place.