Review for Exam 2

Math 2423

March 23, 2016

- Section 5.4: We went through two types of work related problems: 1) the amount of work required to lift a soft, 1-dimensional object such as a chain up by a certain distance, and 2) the amount of work to pump liquid(usually water) in or out of a tank of a specific shape. You should be able to calculate work in both cases. In particular, the "tank" problem usually has some geometric property inherent to the shape of the tank, so that you can write the distance as a function of the location of the "layer" you are analyzing.

- Section 5.5: We defined the average of a continuous function on a closed interval to be the height of the rectangle spanning over that interval representing the area under the graph. You should know how to express it algebraically and evaluate it for a specific function over an interval.

- Section 6.1: We introduced the concept of inverse functions. In particular, the inverse function is only defined for a one-to-one function. To show that a function is one-to-one is a bit tricky, but if a function is not one-to-one, it should be easy to argue either using the definition or by drawing a graph and state how it violates the horizontal line test. Moreover, you are expected to know how to prove the formula for the derivative of the inverse function:

  $$\frac{\mathrm{d}}{\mathrm{dx}} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

  You may refer to the online notes for the proof. If you look up the book, it has a slightly different proof which will also be okay.

- Section 6.1-6.4: In the three sections we introduced the exponential function exp first as an infinite sum of powers of x. Please see the online notes for reference. You should be able to write down our definition of the function exp. The proof for $\exp(x+y) = \exp(x)\exp(x)$ will be too laborious to write down on a test, but we showed $\exp(x) = (\exp(1))^x$ in the homework and this gives us the definition of the natural base e. Moreover, the fact that $\frac{\mathrm{d}}{\mathrm{dx}}\exp(x) = \exp(x)$(or $(e^x)' = e^x$) if you like) is a straightforward consequence of such definition.

  We learned how to differentiate and take the integrals of new functions, given that we know the derivatives of e^x and $\ln x$. For example, you should know how to take the integral of e^x and $\frac{1}{x}$ and use them in a u-substitution.

The differentiation problems are trickier: sometimes you need to take the natural log of both sides of the equation and then differentiate.

- Section 6.5: The natural growth and decay can both be modeled by the exponential function. You should know how to calculate the growth/decay rate given sufficient data, and calculate the half-life of a radioactive matter. You should also be able to predict future quantities given the initial condition and growth/decay rate. All important numbers you will need as part of the calculation will be given.