Solutions to Study Guide

1 LESSON 18: VECTORS

1. A.
$\|\mathbf{u}\| = \sqrt{(-4)^2 + (-8)^2} = 4\sqrt{5}$,
reference angle $\theta = tan^{-1}(8/4) = 63.43°$
u is in the third quadrant, $\alpha = 180° + \theta = 243.43°$

A1.
$\|\mathbf{u}\| = \sqrt{(-24)^2 + (7)^2} = 25$,
reference angle $\theta = tan^{-1}(7/24) = 16.26°$
u is in the second quadrant, $\alpha = 180° - \theta = 163.74°$

2.B.
$\mathbf{u} =< -1, 9 > - < 3, -2 >=< -4, 11 >$

3.C.
i) $5\mathbf{u} = 5 < 4, -3 >=< 20, -15 >$
ii) $\|2\mathbf{v}\| = \|2 < -1, 5 > \| = \| < -2, 10 > \| = \sqrt{(-2)^2 + 10^2} = \sqrt{104} = 4\sqrt{26}$
iii) $\mathbf{u} + \mathbf{v} =< 4, 3 > + < -1, 5 >=< 3, 8 >$
iv) $\mathbf{v} - 3\mathbf{u} =< -1, 5 > -3 < 4, 3 >=< -1, 5 > - < 12, 9 >=< -13, -4 >$
v) $-7\mathbf{w} + 6\mathbf{v} = -7 < -10, -24 > +6 < -1, 5 >=< 70, 168 > + < -6, 30 >=< 64, 198 >$
vi) $\|\mathbf{u} - 3\mathbf{w}\| = \| < 4, 3 > - < -10, -24 > \| = \| < 14, 27 > \| = \sqrt{14^2 + 27^2} = \sqrt{925}$ (this is a really large radical, it's okay to leave it like that)

4.D.
reference angle $\theta = tan^{-1}(8/4) = 63.43°$

$\mathbf{u}$ is in the third quadrant, directional angle $\alpha = 180^\circ + \theta = 243.43^\circ$

5.
unit vector $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{<24,-7>}{\sqrt{(24)^2+(-7)^2}} = \frac{<24,-7>}{25} =< \frac{24}{25}, -\frac{7}{25} >$

6.E.
The unit vector of $\mathbf{u}$ is given by
$\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{<24,-7>}{\sqrt{24^2+(-7)^2}} = \frac{<24,-7>}{25} =< \frac{24}{25}, -\frac{7}{25} >$

The vector we want is
$19\frac{\mathbf{u}}{\|\mathbf{u}\|} = 19 < \frac{24}{25}, -\frac{7}{25} > =< \frac{456}{25}, -\frac{133}{25} >$

F.
initial point = end point - vector $\mathbf{u}$
$=< -4, 7 > - < 7, -13 > =< -11, 20 >$

G.
Let α be the directional angle of $\mathbf{u}$, β be the directional angle of $\mathbf{v}$, γ be the directional angle of $\mathbf{u}+\mathbf{v}$,

$$\begin{aligned} \mathbf{u} &=< \|\mathbf{u}\| cos\alpha, \|\mathbf{u}\| sin\alpha > \\ &=< 18cos80^\circ, 18sin80^\circ > \\ &=< 3.126, 17.73 > \end{aligned} \tag{1.1}$$

$$\begin{aligned} \mathbf{v} &=< \|\mathbf{v}\| cos\beta, \|\mathbf{v}\| sin\beta > \\ &=< 24cos150^\circ, 25sin150^\circ > \\ &=< -20.78, 12 > \end{aligned} \tag{1.2}$$

$\mathbf{u}+\mathbf{v} =< 3.126, 17.73 > + < -20.78, 12 > =< -17.654, 29.73 >$

$\|\mathbf{u}+\mathbf{v}\| = \sqrt{(-17.654)^2 + 29.73^2} = 34.5765$

reference angle θ (for γ) is $\theta = tan^{-1}(\frac{29.73}{-17.654}) = 59.3^\circ$
$\mathbf{u}+\mathbf{v}$ is in the second quadrant, directional angle $\gamma = 180^\circ - \theta = 120.7^\circ$