# Homework #11 Problems
# MATH 4433 Introduction to Analysis

**1.** Prove that if $f$ is defined and continuous on a closed bounded interval $[a, b]$, then the image $f[a, b]$ is also a closed bounded interval.

*Note:* Recall that the *image* of a set $E$ under a function $f$ is defined to be

$$f(E) = \{f(x) : x \in E\}.$$

**2.** Let $f$ be defined and continuous on an open bounded interval $(a, b)$. What forms can the image $f(a, b)$ take, compared to what happens in the previous problem. Must it be an open bounded interval, for example? (This is exercise #32 in the book. The hint is that there are actually 4 possibilities.)

**3.** Prove that the function $f(x) = x^2 - x$ is uniformly continuous on the interval $(0, 1)$.

**4.** Prove that if $f$ is uniformly continuous on an interval of the form $(a, b)$, then $f$ is bounded on $(a, b)$. Prove that this is false if the function is only assumed continuous on $(a, b)$.