Homework #13 Problems
MATH 4433 Introduction to Analysis

1. Let f be continuous on $[a, b]$ and differentiable on (a, b). Let $f(a) = f(b) = 0$. Prove that if $f(c) > 0$ for some c in (a, b), then there exist points x_1 and x_2 in (a, b) such that

$$f'(x_1) > 0 > f'(x_2).$$

2. Let f be twice differentiable on (a, b) and let there be points $x_1 < x_2 < x_3$ in (a, b) such that $f(x_1) > f(x_2)$ and $f(x_3) > f(x_2)$. Prove that there is a point c in (a, b) such that $f''(c) > 0$.

3. Suppose that f is continuous and increasing on $[a, b]$. Prove that $\sup f(E) = f(\sup E)$ for every nonempty set $E \subseteq [a, b]$.

4. Evaluate $\lim\limits_{x \to 0^+} x \ln x$.

5. Use the Inverse Function Theorem to prove that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\arctan x) = \dfrac{1}{1 + x^2}$ for all $x \in (-\infty, \infty)$.