

Homework #14 Problems
MATH 4433 Introduction to Analysis

1. Prove that if f is monotone on $[a, b]$, then f is integrable on $[a, b]$.
2. Prove that if $f(x)$ is integrable on $[a, b]$, then $|f(x)|$ is also integrable on $[a, b]$.
3. Prove that $f(x) = x^2$ is integrable on any interval $[a, b] \subseteq [0, \infty)$, using either the definition of integrability or Remark 8.1.
4. **[Positivity of the Integral]** Let f be continuous and nonnegative on $[a, b]$. Suppose there exists $c \in [a, b]$ such that $f(c) > 0$. Prove that

$$\int_a^b f(x) \, dx > 0.$$

Note: Theorem 8.4 (ii) is a start, but is not the whole argument.

5. Use the fundamental theorem of calculus to find $F'(x)$ if

$$F(x) = \int_0^{x^2} \sqrt{1+t} \, dt.$$