

Homework #3 Problems
MATH 4433 Introduction to Analysis

1. Use the definition of a limit to prove $\lim_{j \rightarrow \infty} \frac{\sin j - \cos j}{j} = 0$.

2. Let $\{a_j\}$ be a convergent sequence of *integers* having the limit L . Prove that the sequence is “eventually constant”, i.e. there exists $N \in \mathbb{N}$ such that for all $j > N$, $a_j = L$.

3. Let $\{a_j\}$ be a sequence of nonnegative real numbers that converges to 0 and $\{x_j\}$ is a sequence for which there exists $N \in \mathbb{N}$ such that for all $j \geq N$, $|x_j - L| < a_j$. Prove that $\lim_{j \rightarrow \infty} x_j = L$.