

Homework #4 Problems  
MATH 4433 Introduction to Analysis

1. Prove the following “Limit Location Theorem”: Suppose that  $\{a_j\}$  is convergent and bounded above by  $M$ . Then the limit  $L$  is less than or equal to  $M$ . (In fact, the sequence only needs to be eventually bounded; i.e. there exists  $N \in \mathbb{N}$  such that for all  $j > N$ ,  $a_j \leq M$ .)

2. Suppose the terms of the sequence  $\{a_j\}$  are colored using red, green, and blue, where each sequence element is given exactly one color and each color is used infinitely often. Form the subsequences  $\{a_{j_r}\}$  = consisting of all the red-colored elements,  $\{a_{j_g}\}$  = all the green-colored elements, and  $\{a_{j_b}\}$  = all the blue-colored elements. Prove that if each of the subsequences formed by the colors converges to the same limit  $L$ , then  $\{a_j\}$  converges to  $L$ .

3. Suppose that  $\{a_j\}$  converges to  $L$  and  $a_j \geq 0$  for all  $j \in \mathbb{N}$ . Prove that the sequence  $\{\sqrt{a_j}\}$  converges to  $\sqrt{L}$ . (Hint: start by proving the statement for  $L = 0$ .)

4. Suppose that  $a_1 \geq 0$  and we define the sequence  $\{a_j\}$  recursively by

$$a_{j+1} = \sqrt{2 + a_j}.$$

For example, let  $a_1 = 7$ . Then the sequence starts out:

$$7, 3, \sqrt{5}, \sqrt{2 + \sqrt{5}}, \dots$$

Prove that if  $\{a_j\}$  converges, the limit  $L$  must be equal to 2, regardless of the starting value  $a_1$ . (Hint: Don't use the definition of the limit, use #1, 3 from this worksheet!)