

Homework #8 Problems
MATH 4433 Introduction to Analysis

Before you start these problems, make a list of the convergence tests and other properties of infinite series.

1. Prove the three parts of the Limit (or Asymptotic) Comparison Test. In each part, let $a_j \geq 0$ and $b_j > 0$ for all j .

a) If $\lim_{j \rightarrow \infty} \frac{a_j}{b_j} = L$, where $0 < L < \infty$, then $\sum_j a_j$ converges if and only if $\sum_j b_j$ converges.

(Hint: Use the Comparison Test.)

b) If $\lim_{j \rightarrow \infty} \frac{a_j}{b_j} = 0$ and $\sum_j b_j$ converges, then $\sum_j a_j$ also converges. (Hint: Start with the Zero Test.)

c) If $\lim_{j \rightarrow \infty} \frac{a_j}{b_j} = \infty$ and $\sum_j b_j$ diverges, then $\sum_j a_j$ also diverges.

2. Determine whether or not each of the following series converges or diverges. Explain which test(s) you use in each.

a) $\sum_{j=1}^{\infty} \frac{j^{\frac{1}{j}}}{j}$

b) $\sum_{j=1}^{\infty} \frac{3j^2 - \sqrt{j}}{j^4 - j^2 + 1}$

c) $\sum_{j=1}^{\infty} \left(\frac{3 + (-1)^j}{3} \right)^j$

3. Determine whether or not each of the following series converges absolutely, converges conditionally, or diverges. Explain which test(s) you use in each.

a) $\sum_{j=1}^{\infty} \frac{(-1)^j j^2}{2^j}$

b) $\sum_{j=1}^{\infty} \frac{(-1)^j \sqrt{j}}{j+1}$