Homework #1 Problems
MATH 4443/5443 Introduction to Analysis

In the following problems, you can use limit laws and the continuity of rational, exponential, and trigonometric functions without proof.

1. Find the pointwise limit of of the sequence of functions $\{f_n\}_{n\in\mathbb{N}}$ where $f_n(x) = \dfrac{x^n}{1+x^n}$ on the domain $x \geq 0$.

2. Find the pointwise limit of of the sequence of functions $\{g_n\}_{n\in\mathbb{N}}$ where $g_n(x) = [\cos(\pi x)]^{2n}$ for $x \in \mathbb{R}$.

3. Prove that the convergence in #1 is uniform on any interval $[0, A]$ where $0 < A < 1$ but is not uniform on $[0, 1]$.

4. Prove that the sequence of functions $\{f_n\}_{n\in\mathbb{N}}$ where $f_n(x) = xe^{-nx}$ converges uniformly on $[0, \infty)$. What function does it converge to?