Homework #2 Problems
MATH 4443/5443 Introduction to Analysis

1. Prove that if $\{f_n\}$ is a sequence of uniformly continuous functions on a domain S which converges uniformly to a function f on S, that f is also uniformly continuous on S. (You may need to remind yourself of the definition of a uniformly continuous function on S.)

2. How do the functions on $(0, 1]$

$$f_n(x) = \begin{cases} n & 0 < x < \frac{1}{n} \\ \frac{1}{x} & \frac{1}{n} \leq x \leq 1 \end{cases}$$

illustrate the result in #1? (Do the hypotheses hold? Does the result follow?)

3. Prove that if $\displaystyle\sum_{n=1}^{\infty} a_n$ is an absolutely convergent series, then $\displaystyle\sum_{n=1}^{\infty} a_n \sin(nx)$ is uniformly convergent on $\mathbb{R}$.

4. Prove that $\sum_{n=1}^{\infty} e^{-nx} \sin x$ converges to a continuous function on $(0, \infty)$. (Note: The convergence is not uniform on $(0, \infty)$.)