

Homework #3 Problems
MATH 4433 Introduction to Analysis

1. Prove that for any choice of c , the Taylor series for $\sin x$ around c converges to $\sin x$ for all x .

2. Determine the interval of convergence of the power series: $\sum_{n=0}^{\infty} \frac{n^n}{n!} x^n$.

3. Let $\sum_{n=0}^{\infty} a_n(x - c)^n$ be a power series. By Lemma 10.1 in the book, let

$$A = \limsup_{n \rightarrow \infty} |a_n|^{1/n}$$

and then let

$$R = \begin{cases} 0 & \text{if } A = \infty \\ \frac{1}{A} & \text{if } 0 < A < \infty \\ \infty & \text{if } A = 0 \end{cases}$$

(The proof uses the Root Test.)

Find the interval of convergence for the power series $\sum_{n=0}^{\infty} ((-1)^n + 3)^n (x - 1)^n$.

4. Compute the Taylor series for $f(x) = \frac{1}{(1-x)^2}$ and find the radius of convergence without needing to actually compute its derivatives.