Homework #8 - due Wed., April 21
MATH 4443/5443

1. Prove that every open set in $\mathbb{R}^n$ is a countable union of open balls $N_{\epsilon_j}(x_j)$.

2. Prove that the Bolzano-Weierstrass property does not hold for $C[a,b]$, the continuous functions on the interval $[a,b]$ with the supremum metric.

3. (Extreme Value Theorem) Prove that if E is compact and nonempty in metric space (X,d) and if $f : E \to \mathbb{R}$ is continuous, then the values

$$M = \sup_{x \in E} f(x)$$

and

$$m = \inf_{x \in E} f(x)$$

are both finite real numbers. Further, prove that there exist x_M and x_m in E such that $f(x_M) = M$ and $f(x_m) = m$.

4. Let E be a compact subset of metric space (X,d) and let $f, g : E \to \mathbb{R}$ be uniformly continuous on E. Prove that the functions $f+g$ and fg are uniformly continuous on E. Where did the compactness of E come in?