Calculus III Honors Spring 2010
Homework 11
Due: Fri. Apr 30, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may *not* use a calculator (or computer) except where stated. Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Reading

Review Exam 2, Chapter 13.

Written Assignment

Total: 100 points. Each problem is worth 5 points unless otherwise noted.

Problem A* (10 pts) Suppose that a zipper manufacturer makes has a probability defect distribution of

$$P(x) = \frac{10}{\sqrt{2\pi}} e^{-50x^2},$$

so that the percentage of zippers produced with defect $< d$ is $\int_{-d}^{d} P(x)dx$.

(i) Compute $\int P(x)dx$ as a series

(ii) If the manufacturer has to throw away any zippers with defect $> \frac{1}{4}$, approximate the percentage $(1 - \int_{-1/4}^{1/4} P(x)dx)$ of zippers thrown away using your answer for (i).

Problem B* (10 pts) Suppose $P(t)$ represents the number of kangaroos (in hundreds) in your backyard after t years with $P(0) = 1$. A simplistic model for $P(t)$ would be $P'(t) = rP(t)$ where r is the population growth rate (whose solution is $P(t) = P(0)e^{rt}$). This does not take into account any conditions, such as the fact that your backyard can only realistically support a population of 250 kangaroos. If the growth rate is $r = \frac{1}{4}$ slightly better model for this is

$$P'(t) = \frac{1}{4}P(t) - \frac{1}{10}P(t)^2.$$

(i) Writing $P(t) = c_0 + c_1 t + c_2 t^2 + \cdots$, use the initial condition $P(0) = 1$, solve for c_0, c_1, c_2 and c_3. (Hint: the series for $P(t)^2$ is

$$P(t)^2 = (c_0 + c_1 t + c_2 t^2 + \cdots)(c_0 + c_1 t + c_2 t^2 + \cdots) = c_0^2 + 2c_0 c_1 t + (2c_0 c_2 + c_1^2)t^2 + \cdots)$$

(ii) Using (i), approximate $P(5)$, i.e., the number of kangaroos (in hundreds) you will have in 5 years.

Section 13.1: 13, 33, 34

Section 13.2: 7, 19, 24

Section 13.3: 5, 7, 17*, 27

Section 13.4: 5, 22, 29

Section 13.5: 3, 23, 33 (for 33, find a vector equation $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0})$ and an equation in terms of x, y, z coordinates)

Starred (*) problems mean problems for which you are allowed (and should) use a calculator.