Here are some supplementary practice problems for the final exam. **Warning: they are not meant to be a comprehensive set of review questions.** In particular, I tried to avoid too much repetition from Exams 1 and 2, the review problems for Exam 2, and the final homework (HW 7). You should definitely make sure you are comfortable with those problems as well.

1. Give an example of a sentence which is not a statement.
2. Prove or disprove: every function from $A = \{1, 2\}$ to $B = \{1, 2, 3\}$ is injective.
3. Prove or disprove: no function from $A = \{1, 2\}$ to $B = \{1, 2, 3\}$ is surjective.
4. Show $x^2 = 1 - x^4$ has no solutions in $\mathbb{Z}$.
5. Show that if x is irrational, so is $\sqrt{x}$.
6. Prove that an integer is divisible by 2 if and only if its last digit is.
7. How many relations are there on $\{a, b, c, d\}$?
8. How many reflexive relations are there on $\{a, b, c, d\}$?
9. How many equivalence relations are there on $\{a, b, c, d\}$? (For in class, just do this for $\{a, b, c\}$.)
10. What is the coefficient of $x^{97}y^3$ in $(x + y)^{100}$?
11. Negate the statement: if $x^2 > 1$, then $x > 1$. Is this statement or its negative true?
12. Let A, B be sets in a universal set X. Prove or disprove: $\overline{A} \cup \overline{B} = \overline{A \cup B}$.
13. Show $a \in \mathbb{Z}$ is odd if and only if $a^2 + 2a + 3$ is even.
14. Prove $n^2 \leq n^3$ for all $n \in \mathbb{N}$.
15. Prove $3^n \geq 2^n + 1$ for all $n \in \mathbb{N}$.
16. Prove or disprove: if A and B are sets, then $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$.
17. Consider a 5-card hand dealt from a standard 52-card deck. How many hands are there such that:

    (a) there are at least 2 cards from 1 suit?

    (b) there are at least 2 cards which are clubs?

    (c) all cards are clubs?

    (d) all cards are clubs but non-consecutive? (a flush in clubs, but not a straight flush—recall if your cards are 2 3 ... 10 J Q K A, then you can think of J as 11, Q as 12, K as 13 and A can be either 1 or 14)
18. Prove or disprove: if A, B, C, D are sets, then $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

19. Prove $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$.

20. Give 2 infinite sets with the same cardinality, and 2 infinite sets with different cardinalities. For the 2 sets with the same cardinality, prove they have the same cardinality.

21. Explain Russell's paradox. What does it mean for set theory?