Here are some comments about how to do these problems/where to find them in the book.

Questions:

1. Find and prove a formula for the alternating sum of the n-th row of Pascal's triangle.

   Explained in class

2. Find and prove a formula for the sum of the first n rows of Pascal's triangle.

   Explained in class

3. Prove $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$ for $n \in \mathbb{N}$.

   Explained in class. See Ch 10, Ex 24

4. Prove $F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$, where F_n is the n-th Fibonacci number (initialized so $F_1 = F_2 = 1$).

   Use induction. See Ch 10, Ex 27

5. Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

   Use induction. See Ch 10, Ex 4

6. Prove that $\gcd(n, n+1) = 1$ for all $n \in \mathbb{N}$.

   Suggested method: contradiction. See Ch 7, Ex 33.

7. Prove the $6|(n^3 - n)$ for any $n \in \mathbb{N}$.

   Suggestion: factor $n^3 - n = n(n+1)(n-1)$. At least one of these factors is even and at least one is divisible by 3 (can prove formally by cases). Or use inductions. See Ch 10, Ex 13.

8. Prove that for all $n \in \mathbb{N}$, $4 \nmid (n^2 + 2)$.

   Can do by cases or contradiction. See Ch 6, Ex 17

9. Prove that $\{4k + 5 : k \in \mathbb{Z}\} = \{4k + 1 : k \in \mathbb{Z}\}$.

   Suggested method: show $A = B$ by showing $A \subseteq B$ and $B \subseteq A$. See Ch 8, Ex 26.

10. How many ways are there to order the letters ABCDE such that

    (a) the two vowels are not adjacent? or

    (b) all three consonants are not in a row?

    (a) 72; (b) 84. My method: count the complement. Here's (a): There are $5! = 120$ total arrangements of the letters. There are two ways vowels can be adjacent (AE or EA). For either of these ways, there are $4! = 24$ such arrangements, i.e., the permutations of the 4 objects AE, B, C, and D. Hence we get $120 - 2 \cdot 24 = 72$. (b) is similar and you get $120 - 3! \cdot 3! = 120 - 36 = 84$.