Elliptic Curves: Problem Set 3 (due Fri Apr 7)

Notation: k denotes a perfect field.

Topics: Elliptic curves over $\mathbb{C}$, group cohomology

1. Exercise III.3.24.
2. Let $E : y^2 = x^3 + ax + b$ be an elliptic curve with $a, b \in \mathbb{Q}$, and $n \geq 1$. Let K be the minimal field of definition for (x, y-coordinates of) all points in $E(\mathbb{C})[n]$. Show $K/\mathbb{Q}$ is a finite Galois extension.
3. Let $E = \mathbb{C}/\Lambda$ be an elliptic curve. Suppose that there exists $\alpha \in \mathbb{C} - \mathbb{Z}$ such that $\alpha\Lambda \subset \Lambda$. Show that α is a root of $x^2 + cx + d$ for some $c, d \in \mathbb{Z}$ such that $c^2 < 4d$. Conclude that $\mathbb{Q}(\alpha)$ is an imaginary quadratic extension. In this case we say E has *complex multiplication* (CM).
4. Complete the proof of Prop IV.1.5.
5. Prove Lemma IV.2.2.