Abstract

Let F be a number field and $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_n(\mathbb{C})$ be a continuous, irreducible representation. Artin conjectured that if ρ is non-trivial, then the associated L-function $L(s, \rho)$ is entire. Langlands generalized this conjecture by asserting that there should be a cuspidal automorphic representation π of $\mathrm{GL}_n(\mathbb{A}_F)$ such that $L(s, \rho)$ and $L(s, \pi)$ agree at almost all places. If such a π exists, ρ is said to be *modular*. Langlands's conjecture does indeed imply Artin's conjecture.

We consider in the thesis the case where ρ is a four-dimensional representation of *solvable type*, i.e., the image of ρ is solvable. We study what is known about Artin's and Langlands's conjectures for ρ. Artin's conjecture is already known in the imprimitive cases, but not in the primitive ones. We show in two new cases, one primitive and one monomial, that ρ is modular; the former case yields new instances of Artin's conjecture. We show that there are only two other primitive cases where one does not know Langlands's conjecture for ρ, and that these cases are symplectic and would follow from certain instances of non-normal quintic base change for GL_4. Our new monomial case is non-essentially-self-dual. In fact we show that if ρ is monomial and essentially self-dual, then it is modular.

We have two other small results for representations in other dimensions. First, if ρ is primitive and three dimensional, then in certain cases we show that the associated eight-dimensional representation $\mathrm{Ad}(\rho)$ is modular. Second, we show that ρ of dimension n having supersolvable image is modular if $n = 2^j$ or $n = 2^j \cdot 3$ for some j.

Lastly, we include in an appendix a proof of Ramakrishnan that if ρ corresponds to π as above, then the complete L-functions for ρ and π are equal as Euler products over $\mathbb{Q}$. More precisely, $L(s, \rho_v) = L(s, \pi_v)$ at every finite place, and $\prod_{v|\infty} L(s, \rho_v) = \prod_{v|\infty} L(s, \pi_v)$.