Name:_____

Math 165 Section 5915 *Practice Exam 1* *September 14, 2010*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let $f(x) = \sqrt{x}$ and $g(x) = 3x + 2$. Find the compositions $f \circ g$ and $g \circ f$ and find their respective domains.

2. Let $f(x) = \cos x$ and $h(x) = \dfrac{2x}{1 + 2x}$. Find the compositions $h \circ f$ and $f \circ h$ and find their domain.

3. Use the table to evaluate each expression: (a) $f(g(1))$, (b) $g(f(1))$, (c) $f(f(1))$, (d) $g(g(1))$, (e) $(g \circ f)(3)$, and (f) $(f \circ g)(6)$

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	1	3

For problems 4-8, find the limit of the function if it exists.

4. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

5. $\lim_{x \to 2} \dfrac{x^2 - x + 6}{x - 2}$

6. $\lim_{x \to -4} \dfrac{x^2 + 5x + 4}{x^2 + 3x - 4}$

7. $\lim_{h \to 0} \dfrac{(2+h)^4 - 16}{h}$

8. $\lim_{x \to 0} \dfrac{\sqrt{1+x^2} - 1}{x^2}$

9. If $4x - 10 \leq f(x) \leq 3x^2 - 5x - 4$ for $x \geq 0$, then find $\lim_{x \to 2} f(x)$

10. Prove that $\lim_{x\to 0} \sqrt{x}\sin\left(\frac{42}{\sqrt[3]{x}}\right) = 0$.

11. Let $f(x) = \begin{cases} 2x-1 & \text{if } x < a \\ x^2 & \text{if } x > a \end{cases}$. For what value of the constant a is the function f continuous on $(-\infty, \infty)$?

12. Find numbers a and b such that $\lim_{x\to 0} \dfrac{\sqrt{ax+b}-2}{x} = 1$

For problems 13 and 14 find the derivative of the following functions using the definition of the derivative.

13. $g(x) = x^2 + 2x + 5$

14. $h(t) = t^3 + 2t - 1$

For problems 15-10, differentiate the following functions with respect to the indicated independent variable.

15. $f(x) = 4x^5 + 7x^4 - 2x^3 + 10x^2 + \pi x - 5$

16. $y(t) = \dfrac{t}{1 - t^2}$

17. $u(x) = (2x^3 + 3)(x^4 - 2x)$

18. $h(w) = \dfrac{2w^2 + 5}{w^5 - 2w + 6}$

19. $v(s) = \left(\dfrac{2}{3s^4} - \dfrac{4}{s^2 + 1}\right)(s^2 + 3)$

20. $g(r) = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2$

21. $w(y) = \sqrt[7]{3y^5} + 3\sqrt{y^7}$

For problems 21-23, find the equation of the tangent line of the following functions at the indicated point.

22. $y = x^5 + 2x^4 - 3x + 2, \text{ at } (0, 2)$

23. $y = x + \dfrac{1}{x}, \text{ at } \left(1, \dfrac{3}{2}\right)$

24. $u(x) = \dfrac{x}{x^2 - 1}$, at $\left(2, \dfrac{2}{3}\right)$

25. Given that $\lim\limits_{x \to 2} (14 - 5x) = 4$, use the precise definition of a limit to find a δ that corresponds to $\varepsilon = 1$.

26. Let $f(x) = \begin{cases} x \cos\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if x} = 0 \end{cases}$. Determine if $f'(0)$ exists and if so compute it. (Hint: Use the definition of the derivative.)