Name:____________________________

Math 165 Section 5915 | *Practice Exam 2* | *October 14, 2010*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 - 8, calculate the derivative of the function with respect to the indicated independent variable.

1. $f(x) = \sin(\csc x)$

2. $g(r) = r^3 \tan(3r^6 - 9r + 2)$

3. $y = \cot(1 + 5x^7)$

4. $h(p) = \dfrac{\sqrt{p^4 + 2p - 7}}{\sin 6p}$

5. $h(\theta) = \dfrac{\tan(6\theta)}{7 - \cos(2\theta)}$

6. $h(z) = \sqrt[5]{\dfrac{z^2 + \cos z}{z - 3z + \tan z}}$

7. $y = \sin(\sin(\cos x))$

8. $g(y) = y^6 \cos(4y - \sin y)$

9. Let $f(x) = \sin x$. Calculate $f^{(35)}(0)$.

10. Let $f(x) = \cos x$. Compute $f^{(150)}(x)$.

11. Let y be implictly defined in x by $x^3 \cot(y) - \tan(y^2) = 12$. Compute $\dfrac{dy}{dx}$.

12. Let z be implictly defined in t by $\sin t \sin z = \sin(t + z)$. Compute $\dfrac{dz}{dt}$.

13. Let r be implictly defined in y by $r^2 + 3yr^7 = 2y + \sin r$. Compute $\dfrac{dr}{dy}$.

14. Let y be implictly defined in r by $r^2 + 3yr^7 = 2y + \sin r$. Compute $\dfrac{dy}{dr}$.

15. Find the equation of the tangent line of $y = \sin x + \cos^2 x$ at $x = \dfrac{\pi}{3}$.

16. Find the equation of the tangent line of $y = \sin(\sin x)$ at $x = \dfrac{\pi}{2}$.

For problems 17 - 21, evaluate the following limits

17. $\displaystyle \lim_{x \to \infty} \frac{3x+5}{x-4}$

18. $\displaystyle \lim_{y \to \infty} \frac{2-3y^2}{5y^2+4y}$

19. $\lim_{x \to -\infty} x + \sqrt{x^2 + 5x}$

20. $\lim_{x \to \infty} \dfrac{5x + 6}{\sqrt{9x^2 + 4}}$

21. $\lim_{x \to \infty} \sqrt{x} \sin\left(\dfrac{1}{x}\right)$

22. Air is being pumped into a spherical ballon so that its volume increases at a rate of 200 cm^3/s. How fast is the radius of the ballon increasing when the diameter is 40 cm?

23. A particle is moving along the curve $y = \sqrt{x}$. As the particle goes through the point $(9, 3)$, the x-coordinate decreases at a rate of 6 cm/s. How fast is the distance from the particle to the origin changing at this moment?

24. A trough is 12 ft long and its ends have the shape of isosceles triangles that are 4 ft across at the top and have a height of 2 ft. If the trough is being filled with slop at a rate of 14 ft^3/min, how fast is the slop level rising when the slop is 6 inches deep?

25. Two people start from the same point. One walks west at 4 mi/h and the other walks southwest at 3 mi/h. How fast is the distance between the people changing after 45 minutes?

26. Estimate $\sqrt{9.001}$ by using linear approximation.

27. Estimate $\sin\left(\dfrac{\pi}{100000}\right)$ by using linear approximation.

28. Given that $y = \tan x$, compute the differential dy when $x = \dfrac{\pi}{4}$ and $dx = .002$.

29. Compute the differential dy when $y = \dfrac{t^2}{\sqrt{t+2}}$ when $t = 2$ and $dt = -0.03$.

For problems 30 - 36, do a complete curve sketching analysis of the given functions, in otherwords find all critical numbers, critical points, inflection points, all relative extrema, on what intervals the function is increasing/decreasing, on what intervals the function is concave up/down, find all asymptotes if there are any and sketch the curve.

30. $y = x^3 + x$

31. $y = x^4 + 4x^3$

32. $y = (4 - x^2)^5$

33. $f(x) = \dfrac{x^2 - 4}{x^2 - 2x}$

34. $f(x) = \dfrac{1}{x^2 - 9}$

35. $f(x) = \dfrac{x}{x^3 - 1}$

36. $y = x^3 + 6x^2 + 9x$

37. If $f(0) = 21$ and $f'(x) \leq 4$ for $0 \leq x \leq 5$, how large can $f(5)$ possibly be?

38. Show that the equation $x^3 - 15x + 9 = 0$ has at most one root in the interval $[-2, 2]$.

39. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all real numbers x?

40. Suppose $f'(x) = 0$ for all x in (a, b). Show that f must be constant on (a, b), in otherwords show that $f(x_1) = f(x_2)$ for every x_1 and x_2 in (a, b).