Name:_____

Math 165 Section 5915 *Practice Exam 3* *November 16, 2010*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 - 8, evaluate the indefinite or definite integrals.

1. $\int (x^7 + 3x^3 - 4x^2 + x - 1)\ dx$

2. $\int_{-1}^{1} t^4\sqrt{6 + 5t^5}\ dt$

3. $\int w\sqrt{3w + 2}\ dw$

4. $\displaystyle\int 3x\sqrt[3]{2x+1}\,dx$

5. $\displaystyle\int_{-\sqrt[3]{\pi}}^{\sqrt[3]{\pi}} x^2\sin(x^3)\,dx$

6. $\displaystyle\int[-\csc^2\theta+\sin(2\theta)\cos(2\theta)]\,d\theta$

7. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x}\, dx$

8. $\displaystyle\int_1^5 \frac{\pi dy}{y\sqrt{y}}$

9. Let $f(x)$ be a continuous function. If $\displaystyle\int_0^2 f(x)\, dx = 5$ and $\displaystyle\int_2^{12} f(x)\, dx = 89$, then evaluate $\displaystyle\int_0^{12} f(x)\, dx$.

10. Let $g(x)$ be a continuous function. If $\int_1^4 g(x)\ dx = 12$, then evaluate $\int_1^2 xg(x^2)\ dx$.

11. Evaluate $\int_{-2}^{3} f(x)\ dx$, where $f(x) = \begin{cases} x^2 + 2 & x \geq 0 \\ 2 - x & x < 0 \end{cases}$

12. Let $g(x) = \int_0^{4x^7} \frac{1}{\sqrt{1+t^2}}\ dt$. Compute $g'(x)$.

13. Let $f(r) = \displaystyle\int_3^{\cot r} \frac{1+x^7}{\cos x}\, dx$. Compute $f'(r)$.

14. Let $h(y) = \displaystyle\int_{\sin y}^{\cos y} \sqrt{\frac{6x^2+3x-1}{\tan^2(x+1)-\sec x}}\, dx$. Compute $h'(y)$.

15. Find the most general antiderivative of $f(x) = \cos x + 2\sec^2 x + x^3 + 1$.

16. Find an antiderivative of $g(t) = t^3 + 2t^6 - \sec t \tan t + 4$.

17. Let $h''(x) = x^2 + 2x - \cos x$, $h'(0) = 2$ and $h(0) = -3$, find $h(x)$.

18. Using the properties of integrals, prove the following estimate: $\displaystyle\int_0^3 x^3 \cos x \, dx \leq \frac{81}{4}$.

19. Using the properties of integrals, prove the following estimate:

$$\frac{\sqrt{2}\pi}{24} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x \; dx \leq \frac{\sqrt{3}\pi}{24} \; .$$

20. Use Newton's Method to compute x_3 if $x_1 = 1$ to the equation $x^3 + 2x - 4 = 0$.

21. Use Newton's Method to compute x_3 if $x_1 = 1$ to the equation $x^5 - x - 1 = 0$.

22. The velocity of a particle is given by $v(t) = t^2 - 2t - 8$ for $1 \leq t \leq 6$. Find the displacement and the distance traveled by the particle in the given time.

For problems 23 - 28, find the area between the indicated curves.

23. The curves $y = x^2 - 2x$ and $y = x + 4$.

24. The curves $y = 1 + \sqrt{x}$ and $y = \dfrac{3 + x}{3}$.

25. The curves $y = \cos x$ and $y = 2 - \cos x$ between $0 \leq x \leq 2\pi$.

26. The curves $y = x^2$ and $y^2 = x$.

27. The curves $4x + y^2 = 12$ and $y = x$.

28. The curves $y = |x|$ and $y = x^2 - 2$.

For problems 29-35 find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

29. The region bounded by $y = 1 - x^2$ and $y = 0$ about the x-axis.

30. The region bounded by $y = \sqrt{9 - x^2}$, $y = 0$, $x = 1$ and $x = 2$ about the x-axis.

31. The region bounded by $x = y - y^2$ and $x = 0$ about the y-axis.

32. The region bounded by $y = x^2$ and $y = 4$ about the line $y = 4$.

33. The region bounded by $x = y^2$ and $x = 1$ about the line $x = 1$.

34. The region bounded by $y = x$ and $y = \sqrt{x}$ about the x-axis.

35. The region bounded by $y = x^2$ and $y^2 = x$ about the line $y = 2$.

36. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

37. Suppose that Eric the encloser wants to enclose a rectangular region that has area equal to 3000 square feet. How much fencing does he need to buy in order to minimize the cost of the fencing?

38. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.

39. If 1200 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box.

40. Write the integral $\displaystyle\int_0^2 \frac{1}{1+x}\,dx$ as a limit of a Riemann sum.

41. Write the integral $\displaystyle\int_1^3 x^4\,dx$ as a limit of a Riemann sum.

42. Evaulate the following limit $\displaystyle\lim_{n\to\infty}\sum_{i=0}^{n}\frac{i^3}{n^4}$.

43. Evaulate the following limit $\lim_{n\to\infty} \frac{3}{n} \sum_{i=0}^{n} \sqrt{1+\frac{3i}{n}}$.

44. Evaulate the following limit $\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=0}^{n} \sin\left(\frac{i\pi}{n}\right)$.