Name: Solutions

Math M119 Section 22611 *Exam 2* *March 25, 2010*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit! There are 11 problems all them worth 5 points each, except for problem 9 which is worth 10 points for a total of 60 points. There are also a bonus problem at the end worth 3 points.

For problems 1-6 find the derivative of the following functions with respect to the indicated independent variable.

1. $g(x) = x^4 - 6x^3 + 5x^2 - 7$

$g'(x) = 4x^3 - 18x^2 + 10x$

2. $f(t) = (t^4 + 2)(6t - 7)$

$f'(t) = (t^4+2)'(6t-7) + (t^4+2)(6t-7)'$

$= 4t^3(6t-7) + 6(t^4+2)$

3. $c(x) = \dfrac{2-x^2}{4+5x^8}$

$$c'(x) = \frac{(2-x^2)'(4+5x^8) - (2-x^2)(4+5x^8)'}{(4+5x^8)^2}$$

$$= \frac{-2x(4+5x^8) - 40x^7(2-x^2)}{(4+5x^8)^2}$$

4. $y(r) = re^{-8r}$

$$\frac{dy}{dr} = \frac{d}{dr}(r)e^{-8r} + r\frac{d}{dr}(e^{-8r})$$

$$= e^{-8r} + re^{-8r}\frac{d}{dr}(-8r)$$

$$= e^{-8r} - 8re^{-8r}$$

5. $w(x) = \ln(x^3 + 6)$

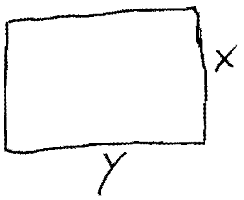
$$w'(x) = \frac{1}{x^3+6} \cdot (x^3+6)'$$

$$= \frac{3x^2}{x^3+6}$$

6. $P(t) = \ln(3) \cdot 3^t + e^3$

$$P'(t) = \ln(3) \cdot 3^t \cdot \ln(3) = (\ln(3))^2 \cdot 3^t$$

7. Suppose that Eric the encloser wants to enclose a rectangular region that has area equal to 1500 square feet. How much fencing does he need to buy in order to minimize the cost of the fencing?



$A = xy = 1500 \Rightarrow y = \frac{1500}{x}$

$P = 2x + 2y$

then $P = 2x + \frac{3000}{x}$ so $P' = 2 - \frac{3000}{x^2}$

and $P' = 0 \Rightarrow 2x^2 = 3000 \Rightarrow x = \pm\sqrt{1500} = \pm 10\sqrt{15}$ but $\cancel{-10\sqrt{15}}$ ~~so~~

so $x = 10\sqrt{15} \approx 38.73$ and $y = \frac{1500}{10\sqrt{15}} = 10\sqrt{15}$ and $P = 20\sqrt{15} + 20\sqrt{15}$
$= 40\sqrt{15} \approx 154.92$

needs 154.92 ft of fencing

8. At a price of \$25 per ticket, the Purdue athletics department can fill every seat in Mackey Arena, which has a capacity of 14,123. For every additional dollar charged, the number of people buying tickets decreases by 75. What ticket price maximizes revenue?

$q(p) = 14{,}123 - 75(p-25)$
$= 14{,}123 - 75p + 1875 = 15{,}998 - 75p$

so $R = pq = p(15{,}998 - 75p) = 15{,}998p - 75p^2$

and $R' = 15{,}998 - 150p = 0 \Rightarrow p = 106.65$

then $R'' = -150$ so $R''(106.65) = -150 < 0 \Rightarrow p = 106.65$

maximizes revenue.

9. Given that $f(x) = x^4 + 4x^3$, sketch the curve, i.e. find all critical points, critical values, inflection points, on which intervals f is increasing and/or decreasing, on which intervals f is concave up and/or down, and all local extrema and draw it.

$f'(x) = 4x^3 + 12x^2$

$= 4x^2(x+3)$

$f''(x) = 12x^2 + 24x$

$= 12x(x+2)$

C.P.

$x = 0, -3$

C.V.

$(0,0)$

$(-3,-27)$

↑ local min and global min

I.P.?

$x = 0, -2$ ✓

$f \nearrow : (-3,0) \cup (0,\infty)$

$f \searrow : (-\infty,-3)$

$f \frown : (-2,0)$

$f \smile : (-\infty,-2) \cup (0,\infty)$

− + +

-3 0

$f'(x) = 4x^2(x+3)$

$f'(-4) = 4(16)(-1) < 0$

$f'(-1) = 4(1)(2) > 0$

$f'(1) = 4(1)(4) > 0$

+ − +

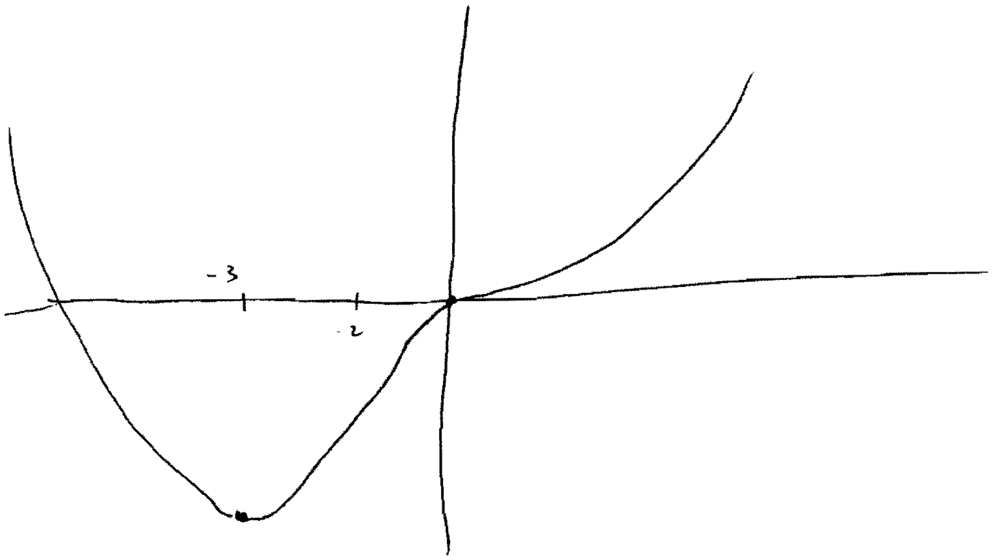
-2 0

$f''(x) = 12x(x+2)$

$f''(-3) = 12(-3)(-1) > 0$

$f''(-1) = 12(-1)(1) < 0$

$f''(1) = 12(1)(3) > 0$

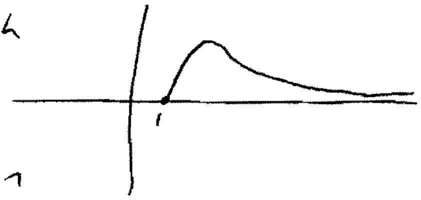


10. Given that $f(x) = \dfrac{\ln(x)}{x}$, find the exact global maximum and/or minimum of f on $x \geq 1$.

$f(1) = \frac{\ln(1)}{1} = 0 \qquad f'(x) = \frac{\frac{1}{x}(x) - \ln x}{x^2} = \frac{1 - \ln x}{x^2} \qquad f' = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

$f''(x) = \frac{-1}{x^5}$ so $f''(e) = \frac{-1}{e^5} < 0 \Rightarrow (e, \frac{1}{e})$ is local max

and $(1, 0)$ is a local min but notice the graph



so as $x \to \infty \Rightarrow f(x) \to 0 \quad \therefore \quad (1,0)$ is a global min

and $(e, \frac{1}{e})$ is a global max

11. Dirac's delicious sushi restaurant has cost and revenue functions, $C(q) = 5q^3 + \dfrac{q}{780} - 908$ and $R(q) = \ln(q) + 5q^3$ respectively. At what quantity sold is the maximum profit achieved?

$\pi(q) = R(q) - C(q) = \ln(q) + 5q^3 - (5q^3 + \frac{q}{780} - 908) = \ln(q) - \frac{q}{780}$

so $\pi'(q) = \frac{1}{q} - \frac{1}{780}$ and $\pi' = 0 \Rightarrow \frac{1}{q} = \frac{1}{780} \Rightarrow q = 780$

then $\pi''(q) = -\frac{1}{q^2}$ and $\pi''(780) = \frac{-1}{(780)^2} < 0 \Rightarrow q = 780$ maximized profit

12. (BONUS) Suppose that $y = f(x) = \dfrac{e^{45}\ln(400) - \frac{e^{78}}{e^{\ln(456e - e^{76})}} + e^3}{\ln(e^7) - e^{56}\ln(6) - 3^{e^2}}$. Find $\left.\dfrac{dy}{dx}\right|_{x=10^{100}}$.

$f(x) = \text{constant} \quad \therefore \quad \frac{dy}{dx} = 0 \quad \therefore \quad \left.\frac{dy}{dx}\right|_{x=10^{100}} = 0$