Department of Mathematical Sciences IUPUI
Math-M 119 A Brief Survey Of Calculus
Midterm Examination Spring 2010

- This exam consists of 5 pages and 17 problems. Make certain you have all five pages before you start.
- Time: 75 minutes
- Scrap paper, notes, books, portable electronic devices, and laptops are not to be used during the exam
- Cell phones should be off. Earpieces other than medically required hearing aids are not permitted.
- **To receive credit show supporting work**
- PRINT your name Key

Do Not Write in this Area		
1	2	
2	2	
3	2	
4(a)	2	
4(b)	2	
5	2	
6(a)	1	
6(b)	1	
6(c)	1	
7	2	
8	2	
9	2	
10	2	
11	2	
12	2	
13	2	
14	2	
15	2	
16	2	
17	2	
TOTAL	37	

1. A company's pricing schedule is given below. Find a formula which expresses p as a **linear** function of q.

q quantity	1	3	5	8
P price/item	70.5	65.5	60.5	53

$p(q) = mq + b$

$m = \frac{65.5 - 70.5}{3 - 1} = -\frac{5}{2} = -2.5$

$p(q) = -2.5q + b$

$70.5 = p(1) = -2.5 + b$

$b = 70.5 + 2.5 = 73$

1. $p(q) = -2.5q + 73$ (2)

2. The quantity of a radioactive substance remaining after t years is given by the formula: $Q(t) = Q_0 e^{(-.0117)t}$ where Q_0 is the initial quantity. Determine the half-life of the substance.

$\frac{1}{2} = e^{-0.0117t}$

$\ln(\frac{1}{2}) = -0.0117t$

$t = \frac{\ln(\frac{1}{2})}{-0.0117} \approx 59.24$

2. 59.24 years (2)

3. You invest money in an account that pays 5 % compounded *continuously*. How much should you invest if you wish the account to be worth $12000 in 10 years?

$P(t) = P_0 e^{kt}$ $\quad k = .05, \; t = 10$

$\Rightarrow \quad 12000 = P_0 e^{(.05)(10)}$

$P_0 = \frac{12000}{e^{.5}} \approx 7278.3679$

2. \$7,278.37 (2)

4. A ball is dropped off a building 500 feet high. Until the ball hits the ground, the ball's height above the ground $s(t)$ (in *feet*) is given by the equation: $s(t) = -16t^2 + 500$ where t is the elapsed time (*seconds*) since the ball was dropped.

(a) Compute the average rate of change $\frac{\Delta s}{\Delta t}$ between $t = 2$ and $t = 3$.

$$\frac{\Delta s}{\Delta t} = \frac{s(3) - s(2)}{3-2} = \frac{-144+500+64-500}{1} = -80$$

4(a). $\frac{\Delta s}{\Delta t} =$ -80 ft/sec (2)

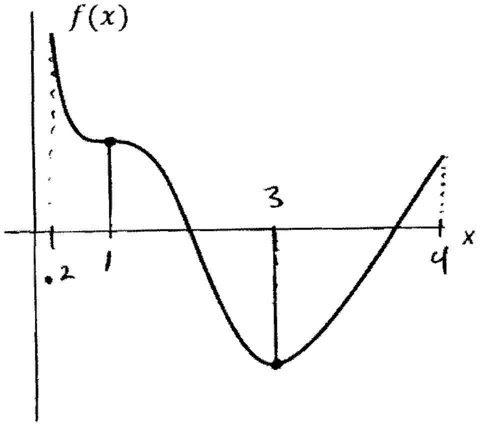
(b) Find the ***instantaneous*** velocity of the ball at time $t = 2$. (An ***exact*** answer, not an estimate)

infs. veloc $= v(t) = s'(t) = -32t$

at $t=2$ $v(2) = -32(2) = -64$

4(b). -64 ft/sec (2)

5. Sketch a reasonable graph of the first derivative $f'(x)$ of the function $y = f(x)$ depicted below. The x-intercepts of your graph should be just where you want them. Your graph should be above (below) the x-axis when you want it to be.



$f' < 0$ on $(.2, 3)$

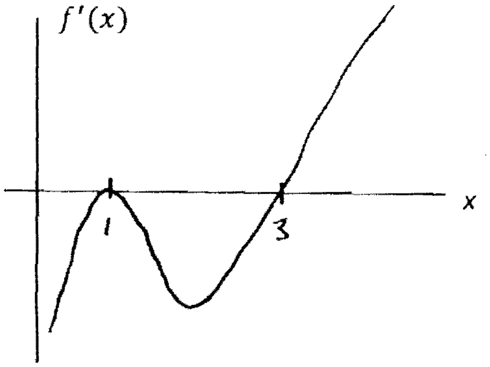
$f' > 0$ on $(3, 4)$

$f'' > 0$ on $(.2, 1) \cup (2, 4)$

so $f' \nearrow$ on $(.2, 1) \cup (2, 4)$

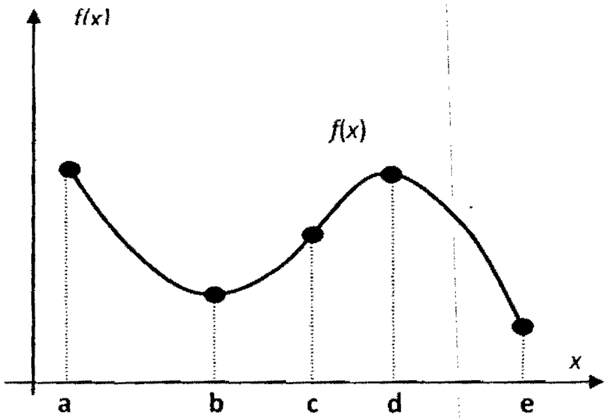
$f'' < 0$ on $(1, 2)$

so $f' \searrow$ on $(1, 2)$



(2)

6. Consider the function $y = f(x)$ depicted below:



(a) When (for which point(s) on the x-axis) is $f'(x) = 0$?

6(a) $x = b, d$ (1)

(b) When (for which interval(s)) is $f'(x) < 0$?

6(b) $(a,b) \cup (d,e)$ (1)

(c) When (for which interval(s)) is $f''(x) < 0$?

6(c) (c,e) (1)

7. For a function $f(x)$ we know that $f(20) = 240$ and $f'(20) = 2$
Use a local linear approximation to estimate $f(18)$.

$f(20) - f(18) \approx f'(20)(20-18)$

$\Rightarrow \quad 240 - f(18) \approx 2(2)$

$\Rightarrow \quad f(18) \approx 240 - 4 = 236$

7. $f(18) \approx 236$ (2)

8. Consider a function defined over the whole real line such that $f'(x) = 3x - 15$.
When (over what interval) is f *increasing?*

$f \nearrow$ means $f' > 0$

so $3x - 15 > 0$

$\Rightarrow 3x > 15$

$x > 5$

8. $(5, \infty)$ (2)

9. Given $y = \sqrt{3}x^7 - \left(\frac{1}{5}\right)x^3$

Find $\frac{dy}{dx}$

$\sqrt{3}$ is just a number

9. $y' = 7\sqrt{3}\,x^6 - \frac{3}{5}x^2$ (2)

10. Find an equation for the tangent line to the curve $y = f(x) = x^3 - 5$ at $x = 2$.

$y' = 3x^2$ so $m = y'(2) = 3(2)^2 = 12$

thru $(2, f(2)) = (2, 2^3 - 5) = (2, 3)$

$y - 3 = 12(x - 2)$

$y = 12x - 21$

10. $y = 12x - 21$ (2)

11. Given $f(t) = \frac{1}{t}$

Find $f'(5)$

$f(t) = t^{-1}$ $f'(t) = -t^{-2} = \frac{-1}{t^2}$

$f'(5) = \frac{-1}{5^2} = \frac{-1}{25}$

11. $f'(5) = \frac{-1}{25}$ (2)

12. Given $y = \sqrt{x}$

Find $\left.\frac{dy}{dx}\right|_{x=49}$

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

so $\left.\frac{dy}{dx}\right|_{x=49} = \frac{1}{2\sqrt{49}} = \frac{1}{2(7)} = \frac{1}{14}$

12. $\left.\frac{dy}{dx}\right|_{x=49} = \frac{1}{14}$ (2)

13. Given $P(t) = 100e^{(.05)t}$
Find $P'(t)$

$$P'(t) = 100\, e^{(.05)t} \cdot (.05)$$
$$= 5e^{.05t}$$

13. $P'(t) = 5e^{(.05)t}$ (2)

14. Given $y = 5^x$ Find $\left.\frac{dy}{dx}\right|_{x=-1}$ (approximate to 4 decimal places please)

$$\frac{dy}{dx} = 5^x \cdot \ln 5$$
$$\left.\frac{dy}{dx}\right|_{x=-1} = 5^{-1} \cdot \ln 5 = \frac{\ln 5}{5} \approx .3218$$

14. $\left.\frac{dy}{dx}\right|_{x=-1} \approx .3218$ (2)

15. Given $f(t) = (t^4 + 1)^{50}$

Find $f'(t)$

$$f'(t) = 50(t^4+1)^{49} \cdot \frac{d}{dt}(t^4+1)$$
$$= 50(t^4+1)^{49} \cdot 4t^3$$
$$= 200t^3(t^4+1)^{49}$$

15. $f'(t) = 200t^3(t^4+1)^{49}$ (2)

16. Given $y = \ln(x^2 + 5)$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{x^2+5} \cdot \frac{d}{dx}(x^2+5)$$
$$= \frac{1}{x^2+5} \cdot 2x = \frac{2x}{x^2+5}$$

16. $\frac{dy}{dx} = \frac{2x}{x^2+5}$ (2)

17. Given $f(z) = z^2 \cdot \ln z$ Find $f'(z)$

$$f'(z) = (z^2)' \ln z + z^2 \cdot (\ln z)'$$
$$= 2z \ln z + z^2 \cdot \left(\frac{1}{z}\right)$$
$$= 2z \ln z + z$$

17. $f'(z) = 2z\ln z + z$ (2)