

For problems 1 and 2 find the radius of convergence and the interval of convergence of the following series:

1.
$$\sum_{n=1}^{\infty} \frac{10^n (x-2)^n}{n^3}$$

2.
$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n (x-3)^n}{2n+1}$$

For problems 3 and 4 use the definition of Maclaurin series to compute the Maclaurin series of the following functions:

3. $f(x) = \cos(\pi x)$

4. $f(x) = e^{-3x}$

For problems 5 and 6, let C be a curve defined by the following parametric equations. Find all values of t for which the curve has horizontal and vertical tangents.

5. $x = e^{\cos t}$ and $y = e^{\sin t}$

6. $x = t^3 - 3t$ and $y = t^2 - 6$

For problems 7 and 8, let C be a curve defined by the following parametric equations. Find the equation of the tangent line at the given point.

7. $x = 1 + \ln t$ and $y = t^2 + 2$ at $(1, 3)$

8. $x = t - t^{-1}$ and $y = 1 + 2t^2$ at $(0, 3)$

For problems 9 and 10, find the exact length of the curve given by the following parametric equations:

9. $x = e^t + e^{-t}$ and $y = 5 - 2t$ on $0 \leq t \leq 3$

10. $x = 3 \cos t - \cos(3t)$ and $y = 3 \sin t - \sin(3t)$ on $0 \leq t \leq \pi$, you will need the identity $\cos(a - b) = \sin a \sin b + \cos a \cos b$.

For problems 11 and 12, find a Cartesian equation for the following polar curves:

11. $r = 5 \cos \theta$

12. $r = \tan \theta \sec \theta$

For problems 13 and 14, find a polar equation for the following Cartesian curves:

13. $x^2 + y^2 = 6x$

14. $x^2 - y^2 = 1$

15. Let $c(t)$ and $s(t)$ be the following functions

$$c(t) = \int_0^t \cos\left(\frac{\pi u^2}{2}\right) du \quad \text{and} \quad s(t) = \int_0^t \sin\left(\frac{\pi u^2}{2}\right) du$$

A curve C is defined parametrically by $x = c(t)$ and $y = s(t)$. Find the exact length of the curve on $0 \leq t \leq a$, for real number $a > 1$. What happens as $a \rightarrow \infty$?