Name:____________________________

Math 2433 Section π | *Practice Final Exam* | *November 8, 2013*

1. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+n}$.

2. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$.

3. Find the radius and interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$.

4. Find the radius and interval of convergence of $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{(x+3)^n}{3^n \ln n}$.

5. Using the definition of Taylor series, find the Taylor series of $f(x) = \ln(3x)$ at $a = 1$.

6. Using the definition of Taylor series, find the Taylor series of $f(x) = \cos(\pi x)$ at $a = 1$.

7. Find the area of one loop of the polar curve $r = 4\sin(2\theta)$.

8. Find the area of one loop of the polar curve $r = 6\cos(3\theta)$.

9. Find the volume of the parellelpiped determined by the three vectors: $\mathbf{a} = \langle 3, 5, 1 \rangle$, $\mathbf{b} = \langle -1, 4, 7 \rangle$, and $\mathbf{c} = \langle 5, 7, 2 \rangle$.

10. Find the volume of the parellelpiped determined by the three vectors: $\mathbf{a} = \langle 1, 2, 3 \rangle$, $\mathbf{b} = \langle 1, 2, 1 \rangle$, and $\mathbf{c} = \langle 1, 1, 1 \rangle$.

11. If $\mathbf{a} \cdot \mathbf{b} = \sqrt{2}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 0, 1 \rangle$, determine the angle between **a** and **b**.

12. If $\mathbf{a} \cdot \mathbf{b} = 10\sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle \sqrt{20}, 4, 8 \rangle$, determine the angle between **a** and **b**.

13. Find an equation of the plane through the three points $P = (1, 4, 2)$, $Q = (-3, 6, 11)$, and $R = (5, 2, -3)$.

14. Find an equation of the plane through the three points $P = (1, 0, 5)$, $Q = (4, -1, 3)$, and $R = (9, 7, -2)$.

15. Find an equation of the tangent line to the curve $\mathbf{r}(t) = \langle 2\sin t, 2\sin(2t), 2\sin(3t)\rangle$ at the point $(1, \sqrt{3}, 2)$.

16. Find an equation of the tangent line to the curve $\mathbf{r}(t) = \langle \sqrt{t^2+3}, \ln(t^2+3), t\rangle$ at the point $(2, \ln 4, 1)$.

17. Find the exact length of the curve $\mathbf{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle$ on $0 \leq t \leq \pi/4$.

18. Find the exact length of the curve $\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$ on $0 \leq t \leq 1$.

19. Find the unit tangent, unit normal, and binormal vectors of $\mathbf{r}(t) = \langle \cos t, \sin t, 10 \rangle$.

20. Find the unit tangent, unit normal, and binormal vectors of $\mathbf{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle$.

21. Find the curvature of the curve with parametric equations:

$$x(t) = \int_0^t \sin\left(\frac{\pi}{2}u^2\right)\,du \text{ and } y(t) = \int_0^t \cos\left(\frac{\pi}{2}u^2\right)\,du$$