

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 - 6, compute the following double integrals:

1. $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) \, dx \, dy$

2. $\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x} \, dx \, dy$

3. $\int \int_R \frac{y^2}{x^2 + y^2} dA$ where $R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4\}$.

4. $\int \int_D \cos \sqrt{x^2 + y^2} dA$ where D is the disk with center the origin and radius 2.

5. $\int \int_D xy^2 dA$ where D is the quarter of the disk in the first quadrant with center the origin and radius 2.

6. $\int \int_D x \, dA$ where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

For problems 7 - 12, compute the following triple integrals:

7. $\int \int \int_E \frac{z}{x^2 + z^2} \, dV$ where $E = \{(x, y, z) : 1 \leq y \leq 4, y \leq z \leq 4, 0 \leq x \leq z\}$.

8. $\int \int \int_E y \, dV$ where $E = \{(x, y, z) : 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}$.

9. $\int \int \int_E \sqrt{x^2 + y^2} \, dV$ where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

10. $\int \int \int_E (x + y + z) \, dV$ where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

11. $\int \int \int_E (x^2 + y^2 + z^2)^2 \, dV$ where E is the ball with center the origin and radius 2.

12. $\int \int \int_E x e^{x^2+y^2+z^2} dV$ where E is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant.

13. The following integral is an improper integral on the cube $[0, 1] \times [0, 1] \times [0, 1]$

$$\int_0^1 \int_0^1 \int_0^1 \frac{1}{1 - xyz} dx dy dz$$

Show that one has the following:

$$\int_0^1 \int_0^1 \int_0^1 \frac{1}{1 - xyz} dx dy dz = \sum_{n=1}^{\infty} \frac{1}{n^3}$$