Name:______________________________

Math 2443 Section 170 | *Practice Exam 3* | *May 5, 2014*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Evaluate the line integral:

$$\int_C 2x \; ds$$

where C : consists of the arc C_1 of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ followed by the vertical line segment C_2 from $(1, 1)$ to $(1, 2)$.

2. Evaluate the line integral:

$$\int_C xy^4 \; ds$$

where C : right half of the circle $x^2 + y^2 = 16$ oriented in the counterclockwise direction.

3. Determine if $\mathbf{F} = \langle (1+xy)e^{xy}, x^2e^{xy} \rangle$ is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

4. Determine if $\mathbf{F} = \langle xy^2, x^2y \rangle$ is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

5. Use Green's Theorem to evaluate the following line integral:

$$\int_C y^2 \, dx + 3xy \, dy$$

where C is the boundary of the semiannular region D in the upper half plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ oriented in the counterclockwise direction.

6. Use Green's Theorem to evaluate the following line integral:

$$\int_C (3y - e^{\sin x})\ dx + (7x + \sqrt{y^4 + 1})\ dy$$

where C: circle $x^2 + y^2 = 9$ oriented in the counterclockwise direction.

7. Compute the curl and divergence of $\mathbf{F} = \langle yze^{xz}, e^{xz}, xye^{xz} \rangle$. Is $\mathbf{F}$ conservative? If so, find a function f such that $\mathbf{F} = \nabla f$.

8. Compute the curl and divergence of $\mathbf{F} = \langle \sin y, x \cos y + \cos z, -y \sin z \rangle$. Is $\mathbf{F}$ conservative? If so, find a function f such that $\mathbf{F} = \nabla f$.

9. Use Stokes' Theorem to evaluate the following surface integral:

$$\int\int_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle xz, yz, xy \rangle$ and S is part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the $xy-$plane oriented up.

10. Use Stokes' Theorem to evaluate the following surface integral:

$$\int\int_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle \tan^{-1}(x^2yz^2), x^2y, x^2z^2 \rangle$ and S is the cone $x = \sqrt{y^2 + z^2}$, $0 \leq x \leq 2$ oriented in the direction of the positive $x-$axis.

11. Use the Divergence Theorem to compute the following surface integral:

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle xy, y^2 + e^{xz^2}, \sin(xy) \rangle$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$.

12. Use the Divergence Theorem to compute the following surface integral:

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle x^2 \sin y, x \cos y, -xz \sin y \rangle$ and S: $x^8 + y^8 + z^8 = 8$.

13. Show that for any two functinons f and g, with continuous second partial derivatives, the following holds: $\Delta(fg) = f\Delta g + g\Delta f + 2\nabla f \cdot \nabla g$, where $\Delta f = \text{div } \nabla f$.