Name:_____

Math 3113 Section 003 *Practice Exam 3* *April 1, 2013*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let

$$A = \begin{pmatrix} 6 & 12 \\ 8 & 3 \end{pmatrix}, \quad B(t) = \begin{pmatrix} t & t^2 \\ 1 & t \end{pmatrix}, \text{ and } C = \begin{pmatrix} 21 & 2 \\ 32 & 1 \end{pmatrix}.$$

Determine if the above matrices have inverses. If they have inverses, compute them.

2. Let

$$A = \begin{pmatrix} 9 & 2 \\ 8 & 3 \end{pmatrix}, \quad B(t) = \begin{pmatrix} e^{3t} & -e^{t} \\ e^{-t} & e^{-3t} \end{pmatrix}, \text{ and } C = \begin{pmatrix} 6t & 4 \\ 7 & \frac{1}{t} \end{pmatrix}.$$

Determine if the above matrices have inverses. If they have inverses, compute them.

3. Let

$$A(t) = \begin{pmatrix} \cos t & \tan t \\ -\cot t & \sec t \end{pmatrix} \text{ and } B(t) = \begin{pmatrix} 6e^{5t} & t^5 \\ t^{-5} & e^{-5t} \end{pmatrix}$$

Compute $A'(t)$, $B'(t)$, and $\det(AB)$.

4. Let

$$A(t) = \begin{pmatrix} te^t & \sin t \\ \csc t & 6e^{-t} \end{pmatrix} \text{ and } B(t) = \begin{pmatrix} 3e^t \ln t & \ln t \\ 5 & e^{-t} \end{pmatrix}$$

Compute $A'(t)$, $B'(t)$, and $\det(AB)$.

5. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 2x_1 + 3x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$$

6. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 9x_1 + 5x_2 \\ x_2' = -6x_1 - 2x_2 \end{cases}$$

7. Find the general solution to the following first order system:

$$\begin{cases} x_1' = x_1 - 5x_2 \\ x_2' = x_1 - x_2 \end{cases}$$

8. Find the general solution to the following first order system:

$$\begin{cases} x_1' = -3x_1 - 2x_2 \\ x_2' = 9x_1 + 3x_2 \end{cases}$$

9. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = y(1) = 0 \end{cases}$$

10. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = y'(1) = 0 \end{cases}$$

11. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + \lambda y = 0 \\ y(-\pi) = y(\pi), \quad y'(-\pi) = y'(\pi) \end{cases}$$

12. Let

$$A(t) = \begin{pmatrix} e^t & e^{-t} \\ -e^t & e^t \end{pmatrix} .$$

Compute the eigenvalues and eigenvectors to $A(t)$.