Name: ____________________

Math 4163 Section 002 — *Practice Exam 1* — *January 30, 2014*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Solve the following PDE subject to the following boundary and initial conditions:

$$\begin{cases} u_t = ku_{xx} \\ u_x(0,t) = u_x(L,t) = 0 \\ u(x,0) = f(x) \end{cases}$$

You do not need to derive what the coefficients are in the resulting series solution, just state the formulas for them.

2. Solve the following PDE subject to the following boundary and initial conditions:

$$\begin{cases} u_t = ku_{xx} \\ u(-L,t) = u(L,t) \text{ and } u_x(-L,t) = u_x(L,t) \\ u(x,0) = f(x) \end{cases}$$

You do not need to derive what the coefficients are in the resulting series solution, just state the formulas for them.

3. Solve the following PDE subject to the following boundary and initial conditions:

$$\begin{cases} u_t = k u_{xx} \\ u(0,t) = u(L,t) = 0 \\ u(x,0) = f(x) \end{cases}$$

You do not need to derive what the coefficients are in the resulting series solution, just state the formulas for them.

4. Solve the following PDE subject to the following boundary and initial conditions:

$$\begin{cases} \Delta u = 0 \\ u(0,y) = u(L,y) = u(x,H) = 0 \\ u(x,0) = f(x) \end{cases}$$

You do not need to derive what the coefficients are in the resulting series solution, just state the formulas for them.

5. Compute the Fourier Series for the following function $f(x) = \begin{cases} 0 \text{ for } x < \dfrac{L}{2} \\ x \text{ for } x > \dfrac{L}{2} \end{cases}$

6. Compute the Fourier Series for the following function $f(x) = \begin{cases} -1 \text{ for } x < \dfrac{L}{2} \\ 1 \text{ for } x > \dfrac{L}{2} \end{cases}$

7. Show the following PDE subject to the following boundary and initial conditions has a unique solution:

$$\begin{cases} u_t = ku_{xx} \\ u(-L,t) = T_1(t) \text{ and } u_x(L,t) = T_2(t) \\ u(x,0) = f(x) \end{cases}$$

HINT: First suppose that there are two different solutions, u_1 and u_2 and let $w = u_1 - u_2$ and see what the resulting PDE is. Then consider the energy of w that is $E(t) = \int_{-L}^{L} w(x,t)^2 \, dx$

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9. Let

$$u(x,t) = \frac{1}{\sqrt{4\pi kt}} \int_{-\infty}^{\infty} f(y) e^{-\frac{(x-y)^2}{4kt}} \, dy$$

Show that $u_t = k u_{xx}$ HINT: Just compute the derivatives and see what happens.